

PDEs on the Sphere 2017



Book of abstracts

First author	page
Aranami, K.	3
Arpaia, L.	4
Baldauf, M.	5
Bauer, W.	6
Bauer, W.	7
Benacchio, T.	8
Bénard, P.	9
Borchert, S.	10
Brachet, M.	11
Bui, T.	12
Chandrasekhar, P.	13
Chen Y.	15
Chun, S.	16
Colavolpe, C.	17
Diamantakis, M.	18
Eldred, C.	19
Ferguson, J.	20
Gassmann, A.	21
Hall, D.	22
Jablonowski, C.	23
Jeschke, A.	24
Johansen, H.	25
Kevlahan, N.	26
Knoth, O.	27
Kupiainen, M.	28
Lauritzen, P.	29
Lee, D.	30
Lin, S.-J.	31
Marien, L.	32
McRae, A.	33
Met Office	34
Montanelli, H.	35
Mueller, A.	36
Nair, R.	37
Nam, H.	38
Norman, M.	39
Paldor, N.	40
Park, J.-R.	41
Peixoto, P.	42
Piotrowski, Z.	43
Qaddouri, A.	44
Ramirez, E.	45
Reed, K.	46
Reinecke, A.	47
Sakamoto, M.	48
Schaefer, U.	52
Schreiber, M.	53
Shaw, J.	54
Shipton, J.	55
Shipway, B.	56
Simon, K.	57
Subich, C.	58
Sunkisala, A.	59
Thuburn, J.	60
Toy, M.	61
Vater, S.	62
Waruszewski, M.	63
Weller, H.	64
Yi, T.-H.	65

A time-splitting method for Eulerian based models considering the CFL condition in 3D

Kohei Aranami, Kohei Kawano, Yuji Kitamura, Tabito Hara,
and Junichi Ishida
Japan Meteorological Agency

The Eulerian scheme is one of the common schemes for advection, because this it does not require global communication and is suitable for parallel computers when combined with the horizontally explicit vertically implicit (HEVI) time-splitting approach to deal with fast propagating modes such as sound and gravity waves. In this case, the time-step interval is limited by the Courant-Friedrichs-Lewy (CFL) condition for advection in 3D, i.e.

$$C_x + C_y + C_z \leq C_1, \quad (1)$$

where C_x, C_y, C_z is the Courant number in the horizontal (x, y) and the vertical (z) directions. C_1 is the critical Courant number for one-dimensional case depending on the spatial and temporal discretisations.

Since the aspect ratio is high between the horizontal (x, y) and vertical (z) grid spacing particularly in the lower atmosphere and vertical velocity can grows rapidly associated with development of convective cells in the numerical weather prediction (NWP) models, the vertical Courant number C_z varies more than horizontal Courant number C_x, C_y . Therefore it is important to cope with large C_z appropriately, and one of the approaches is to use a time-splitting method for the vertical advection.

This results in an asymmetric way of time-splitting, i.e. time-splitting is applied in vertical direction while not in horizontal direction, that is suitable for massively parallel computers. A stability analysis based on an idealised two-dimensional advection test case has confirmed that the time-splitting method provides better computational stability. The method has also been incorporated in the Japan Meteorological Agency's limited area model, which makes the robustness of the model significantly better. In the presentation, some details on the time-splitting method, as well as the results of stability analysis on idealised and real data cases will be shown.

An ALE moving mesh method on the sphere for tsunami wave propagation and inundation

L. Arpaia and M. Ricchiuto
INRIA Bordeaux, team CARDAMOM

Abstract. The effectiveness of various moving mesh methods for hydraulic and coastal applications has been recently studied. In particular, in the context of tsunami simulations, the interest in moving meshes may lie in the clever redistribution of mesh points during all the stages of a tsunami. Through a proper sensor based on some error estimate, one could track the source distribution, wave shoaling, runup/rundown with fine scale flooding and finally the complex pattern of reflected and trapped waves. However tsunami dynamics could be strongly affected by earth's curvature and current moving mesh strategies must be carefully reviewed. In this work we develop a moving mesh method for the Shallow Water (SW) equations on portions of the sphere (i.e. the poles will not be considered). First we rewrite the SW equations on the sphere with respect to an arbitrary moving reference system, called Arbitrary Lagrangian Eulerian (ALE) framework (Stavros et al., *Journal of Theoretical and Applied Mechanics* 28, 2008). Second we present a Well-Balanced Residual Distribution (RD) approximation of the SW equation on the sphere in ALE framework. RD are a promising class of multidimensional upwind schemes for solving conservation laws, and have been already extended, for the steady case, to generalized coordinates by Rossmanith (*Numerical Modeling of space plasma flows* 359, 2013). Third a simple moving mesh PDE allows point movements in order to enhance wave patterns. Numerical tests show that our resulting moving mesh algorithm, can improve the resolution of linear and nonlinear waves on the sphere, using a limited number of mesh points.

The HEVI approach in Discontinuous Galerkin methods

Michael Baldauf
Deutscher Wetterdienst

The Discontinuous Galerkin (DG) method is nowadays quite often used in the computational fluid dynamics community to numerically solve the Euler or Navier-Stokes equations. With a certain retardation this method is more and more inspected for its applicability for numerical simulation models in meteorology, too, and examples of successful implementations arise in several institutions worldwide (at NPS, at NCAR, ...). Whereas a standard approach is based on explicit time integration schemes (the RK-DG method e.g. introduced by Cockburn and others), for weather prediction and climate models an implicit approach is required to treat the usually rather flat grid cells near the ground in a stable manner with reasonably large time steps. Alternatively to the use of a fully 3-dimensional (3D) solver, it is also possible to apply the well known HEVI (horizontally explicit - vertically implicit) approach to reduce the size of the implicit equation system.

The presentation will highlight some aspects of the HEVI approach for relatively low order polynomial bases in DG, in particular in combination with IMEX-Runge-Kutta time integration schemes. Some first results with idealized test cases will be shown. Furthermore, a path for a possible future implementation into the global weather prediction and climate model system ICON (a joint development of the Max Planck Institute for Meteorology and the Deutscher Wetterdienst) will be sketched.

Variational integrators for anelastic and pseudo-incompressible flows and various equations of GFD

Werner Bauer^{1,2} and François Gay-Balmaz¹

Abstract

In this talk we present an overview of structure-preserving variational discretizations of various equations of geophysical fluid dynamics, such as the Boussinesq, anelastic, pseudo-incompressible, and shallow-water equations. We further illustrate the wide applicability of this discretization method (e.g. for Magnetohydrodynamics), which is based on a discrete version of the Euler-Poincaré variational method. We introduce the method in detail for the anelastic and pseudo-incompressible equations, two well-known soundproof approximations of compressible flows. This discretization approach relies on a finite dimensional approximation of the (Lie) group of diffeomorphisms that preserve *weighted*-volume forms. These weights describe the background stratification of the fluid and correspond to the weighed velocity fields for anelastic and pseudo-incompressible approximations. In particular, we identify to these discrete Lie group configurations the associated Lie algebras such that elements of the latter correspond to weighted velocity fields that satisfy the divergence-free conditions for both systems. Defining discrete Lagrangians in terms of these Lie algebras, the discrete equations follow by means of variational principles. We verify the structure-preserving nature of the resulting variational integrators applying two test cases. The spectra of internal gravity waves, emitted by a hydrostatic adjustment process, perfectly reflects the models' dispersion relations. And, shape and advection speed of a rising and a falling bubble match very well reference results in literature. As descending from variational principles, the discussed variational schemes exhibit a discrete version of Kelvin circulation theorem, are applicable to irregular meshes, and show excellent long term energy behavior.

¹Laboratoire de Météorologie Dynamique, École Normale Supérieure/CNRS, Paris, France.
gaybalma@lmd.ens.fr

²Imperial College London, United Kingdom. w.bauer@imperial.ac.uk

A structure-preserving split finite element discretization of the split 1D linear shallow-water equations

Werner Bauer, Jörn Behrens

January 14, 2017

Abstract

We present a locally conservative, low-order finite element (FE) discretization of the covariant 1D linear shallow-water equations written in split form (cf. [1]). The introduction of additional differential forms (DF) that build pairs with the original ones permits a splitting of these equations into topological momentum and continuity equations and metric-dependent closure equations that apply the Hodge-star. Our novel discretization framework conserves this geometrical structure, in particular it provides for all DFs proper FE spaces such that the differential operators (here gradient and divergence) hold in strong form. The discrete topological equations simply follow by trivial projections onto piecewise constant FE spaces without need to partially integrate. The discrete Hodge-stars operators, representing the discretized metric equations, are realized by nontrivial Galerkin projections (GP). Here they follow by projections onto either a piecewise constant (GP0) or a piecewise linear (GP1) space.

Our framework thus provides essentially three different schemes with significantly different behavior. The split scheme using twice GP1 is unstable and shares the same discrete dispersion relation and similar second-order convergence rates as the conventional P1-P1 FE scheme that approximates both velocity and height variables by piecewise linear spaces. The split scheme that applies both GP1 and GP0 is stable and shares the dispersion relation of a conventional P1-P0 FE scheme that approximates the velocity by a piecewise linear and the height by a piecewise constant space with corresponding second- and first-order convergence rates. Exhibiting for both velocity and height fields second-order convergence rates, we might consider the split GP1-GP0 scheme though as stable versions of the conventional P1-P1 FE scheme. For the split scheme applying twice GP0, we are not aware of a corresponding conventional formulation to compare with. Though exhibiting larger absolute error values, it shows similar convergence rates as the other split schemes, but does not provide a satisfactory approximation of the dispersion relation as short waves are propagated much too fast. Despite this, the finding of this new scheme illustrates the potential of our discretization framework as a toolbox to find and to study new FE schemes based on new combinations of FE spaces.

References

- [1] Bauer, W. [2016], A new hierarchically-structured n-dimensional covariant form of rotating equations of geophysical fluid dynamics, *GEM - International Journal on Geomathematics*, **7(1)**, 31–101.

Progress and challenges with GungHo, the Met Office's next generation dynamical core

Tommaso Benacchio, Thomas Melvin, the GungHo team

Driven by exponential growth in the core count of high-performance computing facilities, efforts in numerical weather prediction with prospective resolutions approaching the sub-kilometre scale are aiming at scalability as the primary target for next-generation dynamical cores. Following the GungHo project, work at the Met Office is focusing on the implementation of a three-dimensional non-hydrostatic dynamical core based on a cubed-sphere grid arrangement with a mixed finite element scheme.

The adoption of a more homogeneous grid together with the mimetic properties of the chosen spatial discretization aim to tackle the scalability and conservation issues of the currently operational ENDGame model whilst retaining its accuracy properties. Features of the scheme include a bespoke functional space for the potential temperature that reproduces the C-grid horizontal, Charney-Phillips vertical staggering at lowest polynomial order. For advection the target method is a direction-split COSMIC scheme, and an option for a finite volume method of lines scheme is also available. Time integration is handled by the iterative semi-implicit Helmholtz solver also used in ENDGame. Numerical results to date confirm expected behaviour on two- and three-dimensional tests of nonhydrostatic flows on Cartesian and spherical domains.

The scientific aspects are implemented in algorithms and kernels within an object-oriented modern Fortran infrastructure, while parallelization and flexibility to underlying computing architecture are afforded by a Python engine featuring auto-generation of the parallel layer. Promising related results on experiments with shared and distributed memory parallelism will be shown, and current scientific and computational challenges will be discussed.

Circumventing the pole problem for solving PDEs in spherical coordinates with local algorithms

Pierre Bénard
Météo-France, CNRM/GMAP Toulouse, France

The numerical solution of PDEs in spherical coordinates on latitude-longitude (lat-lon) grids with local algorithms is prone to the so-called "pole-problem", and this avenue for solving PDEs has therefore been largely abandoned, to the benefit of less structured grids and the implicit use of local coordinate systems. Here, we examine whether the pole problem (identified in the 1960s) may be circumvented for reduced lat-lon grids with quasi-uniform physical resolution.

The potential benefits of reduced lat-lon grids in spherical coordinates are attractive : the data grid is semi-structured, allowing an easy and computationally-efficient access to high-order-accurate discrete spatial derivative operators. In view of finite volume discretizations, the cells may be considered as quadrilaterals (in the computational domain) thereby naturally allowing a good balance of the degrees of freedom for all prognostic variables (especially when using a non-staggered grid). The solution of PDEs on lat-lon grids in spherical coordinates is also known not to suffer the pole problem when the (non-local) spectral spherical harmonics technique is used, indicating that the sensitivity to the pole problem deeply depend on the spatial discretization technique, and especially the accuracy near poles.

In the presentation, we will show that it is not conceivably impossible to conciliate the imperatives of very high space accuracy near poles (to avoid the pole problem) and local algorithms (to allow scalability on modern computers). The discretization algorithms have to be carefully and specifically designed near poles to allow an accurate and stable evolution, a task that has not been undertaken in the previous litterature. We will propose analyses allowing to identify and circumvent the pole problem with local algorithms, and present some results of very accurate simulations for simple flows in a dedicated shallow-water model.

Extending the ICON-model to the upper atmosphere in order to study gravity wave dynamics
from the troposphere to the thermosphere

Sebastian Borchert ¹⁾, Günther Zängl ¹⁾, Michael Baldauf ¹⁾,
Guidi Zhou ²⁾, Hauke Schmidt ²⁾, Elisa Manzini ²⁾

1) Deutscher Wetterdienst, Offenbach, Germany (sebastian.borchert@dwd.de)

2) Max-Planck-Institut für Meteorologie, Hamburg, Germany

In climate simulations and numerical weather prediction, there are ongoing efforts to raise the upper model lid, acknowledging possible influences of middle and upper atmosphere dynamics on tropospheric weather and climate. As the momentum deposition of gravity waves (GWs) is responsible for key features of the large scale flow in the middle and upper atmosphere, the upward model extension has put GWs in the focus of atmospheric research. The Max Planck Institute for Meteorology (MPI-M) and the German Weather Service (DWD) have been developing jointly the non-hydrostatic global model ICON (Zängl et al, 2015) which features a new dynamical core based on an icosahedral grid. Extending ICON beyond the mesosphere, where most GWs deposit their momentum, requires modifications such as the relaxation of the shallow-atmosphere and other traditional approximations as well as the implementation of additional physical processes that are important to the upper atmosphere. We would like to present aspects of the model development and first evaluation results. This work is part of the research unit: Multi-Scale Dynamics of Gravity Waves (MS-GWaves: sub-project GWINING, <https://ms-gwaves.iau.uni-frankfurt.de/index.php>), funded by the German Research Foundation.

Zängl, G., Reinert, D., Ripodas, M.P., Baldauf, M., 2015: “The ICON (ICOsahedral Non-hydrostatic) modelling framework of DWD and MPI-M: Description of the non-hydrostatic dynamical core”. *Quart. J. Roy. Met. Soc.*, 141, 563 – 579, doi:10.1002/qj.2378

Numerical approximation of propagation problems on the sphere using a compact scheme

Matthieu BRACHET, IECL, Université Lorraine, Metz

Jean-Pierre CROISILLE, IECL, Université Lorraine, Metz

In this presentation, we continue to investigate a compact scheme introduced in [1, 2] with emphasis on PDEs on the sphere involving rotational and/or shear flows. Our scheme is based on the Cubed Sphere. Each point of the grid carries data for the principal unknown q and the tangential gradient $\nabla_T q$. The approximation in space is fully centered with accuracy close to 4. The numerical diffusion is kept as minimal as possible in order to preserve the accuracy of the scheme, even after a large number of time iterations. This diffusion mainly consists in a high frequency filtering of the 1/-1 mode associated with the grid. Recent progress has focused on the importance of the symmetric form of the filtering. Numerical results will be shown, in particular vortex propagation [3] and shear flows problems [4] for large physical time simulations. These results indicate the interest of the present approach for applications in mathematical climatology. More details can be found in [5].

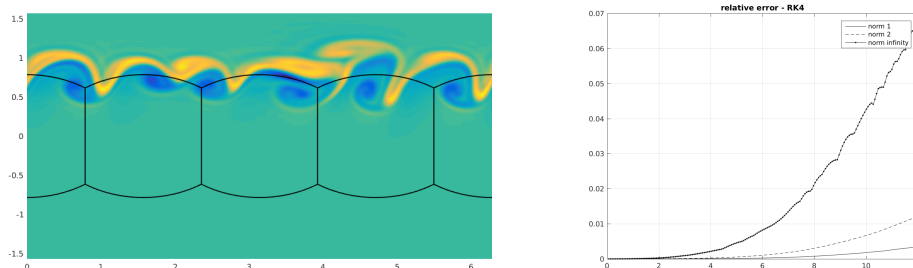


Figure 1: Vorticity of the J. Galewsky et al. test case [4] after 6 days with $6 \times 63^2 + 2$ points on the CS (left), relative error on the advective equation [3] with $6 \times 40^2 + 2$ points (right)

Références

- [1] J.-P. CROISILLE, *Hermitian compact interpolation on the cubed-sphere grid*, Jour. Sci. Comp. , 57, 2013, pp. 193-212.
- [2] J.-P. CROISILLE, *Hermitian approximation of the spherical divergence on the Cubed-Sphere*, J. Comp. App. Maths., 280, 2015, pp. 188-201.
- [3] R. D. NAIR, C. JABLONOWSKI, *Moving Vortices on the Sphere : a test case for horizontal advection problem*, Mon. Wea. Rev. , 136, 2008, pp. 689–711.
- [4] J. GALEWSKY, R. SCOTT, K. RICHARD AND L. M. POLVANI, *An initial-value problem for testing numerical models of the global shallow-water equations*, Tellus A , 56, 2004, pp. 429–440.
- [5] M. BRACHET, J.-P. CROISILLE, *Numerical simulations of propagation problemes on the sphere using a compact scheme*, preprint.

BRACHET Matthieu, Institut Elie Cartan de Lorraine, Université de Lorraine, Site de Metz, Bât. A, Ile du Saulcy, F-57045 Metz Cedex 1

matthieu.brachet@univ-lorraine.fr

CROISILLE Jean-Pierre, Institut Elie Cartan de Lorraine, Université de Lorraine, Site de Metz, Bât. A, Ile du Saulcy, F-57045 Metz Cedex 1

jean-pierre.croisille@univ-lorraine.fr

Some advances in the upwind hybridized discontinuous Galerkin method for dynamical cores

Shinhoo Kang, Sriram Krishnan, Stephen Shannon, Tan Bui-Thanh
Department of Aerospace Engineering, UT Austin

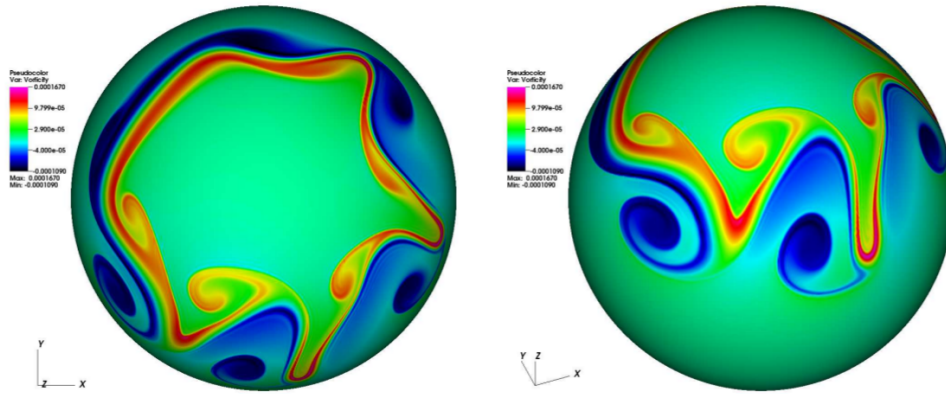
We will present new developments on the emerging Hybridized Discontinuous Galerkin (HDG) method targeting at large-scale and parallel simulation of the dynamical core. In particular, we will present an iterative HDG (iHDG) method that exploits current and future multi-threaded computing system with massive concurrencies. We provide both theoretical justification and numerical results to support the iHDG idea. Furthermore, we also present fast and scalable preconditioning strategies for HDG method that potentially make the HDG approach competitive with the existing methods. Several test cases and models for the dynamical core will be presented to demonstrate the potential of the HDG approach.

DISCONTINUOUS GALERKIN AND SPECTRAL ELEMENT METHODS FOR ROTATING SHALLOW WATER EQUATION ON THE SPHERE

PRAVEEN CHANDRASHEKAR

TIFR Center for Applicable Mathematics, Bangalore, India.
email: praveen@math.tifrbng.res.in

Shallow water equations can be solved either in conservation form (momentum Dv as an independent variable) or in vector-invariant form (velocity v as an independent variable). The primitive form is supposed to be better for consistent vorticity evolution as the curl of this equation gives the vorticity transport equation. However the accurate evolution of vorticity is not automatically guaranteed since vorticity is not directly evolved in time, but must be computed as the curl of velocity. In this work, we propose to evolve vorticity η as an independent variable along with velocity v and depth D , using the vector-invariant form of the shallow water equations. There is a coupling between the velocity and vorticity since the vorticity appears in the Coriolis term of the velocity equation, and the velocity appears in the advective terms of the vorticity equation.

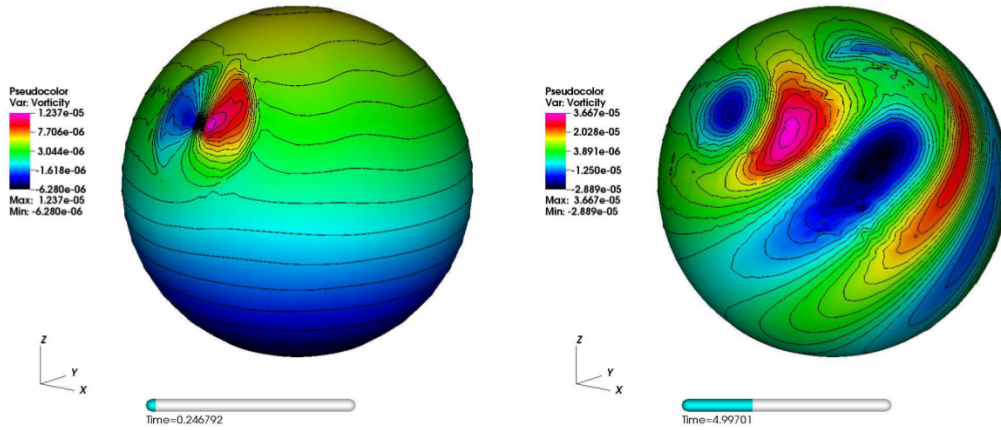


(1) Barotropic instability using DG scheme, solution at 6 days

We develop a discontinuous Galerkin (DG) method on cubed-sphere grid using Cartesian coordinates and tensor product nodal Lagrange basis functions. The numerical flux required by the DG scheme is computed by solving a linearized Riemann problem for the $v - D - \eta$ system. Numerical results on standard test problems show that the scheme has a dissipative character for both total energy and potential enstrophy, while still maintaining low levels of error. Figure (1)

shows the results of the barotropic instability problem of Galewsky et al. on a grid of 6144 cells and degree 6 polynomials (angular resolution of 0.47 deg.). No limiter, filtering or artificial diffusion has been used for this computation.

A second approach we develop is a spectral element method (SEM) using continuous Lagrange basis functions for the vector-invariant form of the equations. We construct a scheme using Cartesian coordinates that conserves total energy for the semi-discrete scheme. However this scheme does not provide good stability for vorticity/enstrophy. Figure (2) shows results of Test 5 from Williamson et al. using 384 cells and degree 5 polynomials (angular resolution of 2.25 deg.), which indicates that vorticity has spurious oscillations. To improve this scheme, we propose to solve vorticity as an independent variable using a DG scheme. The resulting semi-discrete scheme would then conserve total energy but have a dissipative character for vorticity/potential enstrophy. Such a property is desirable since in 2-D flows, energy cascades to large scales while enstrophy cascades to small scales.



(2) Test 5 from Williamson et al. using SEM

An Adaptive Mass Conservative Multi-tracer Efficient Semi-Lagrangian Advection Scheme

Yumeng Chen* and Jörn Behrens

University of Hamburg/CliSAP, Grindelberg 5, 20144 Hamburg, Germany

Abstract: Advection is a dominant process in atmospheric and oceanic dynamics. Adaptive mesh refinement (AMR) is a promising approach for accuracy improvement without the need to uniformly refine the whole domain. We present a novel 2-D adaptive mass conservative multi-tracer-efficient semi-Lagrangian scheme. Our method is based on the flux-form semi-Lagrangian scheme in ECHAM [3]. The goal is to implement AMR only in tracer transport and modify coarse resolution results based on AMR results such that we do not need to change other formulations in ECHAM.

The AMR uniformly divides a cell into four sub-cells and doubles the resolution. To organize the mesh on different refinement level on rectangular grid, a hash table is used [2]. This data structure needs fewer memory and is easier to manage than the widely used tree structure [1].

We will demonstrate how to alleviate problems that arise from hanging nodes by a mixture of 1-D and 2-D schemes. Multi-tracer efficiency is achieved because the same departure position can be used for all tracers in the same time step. Due to the complexity and efficiency needs in climate models, simple mesh refinement criteria are applied and results are shown from several test cases.

References

- [1] Marsha J. Berger and Joseph Oliger. Adaptive mesh refinement for hyperbolic partial differential equations. *Journal of Computational Physics*, 53:484–512, 1984.
- [2] Hua Ji, Fue-Sang Lien, and Eugene Yee. A new adaptive mesh refinement data structure with an application to detonation. 229:8981–8993, 2010.
- [3] Shian-Jiann Lin and Richard B. Rood. Multidimensional Flux-Form Semi-Lagrangian Transport Schemes. *Monthly Weather Review*, 124:2046–2070, 1996.

*yumeng.chen@uni-hamburg.de

METHOD OF MOVING FRAMES TO SOLVE THE SHALLOW WATER EQUATIONS ON ARBITRARY ROTATING CURVED SURFACES

SEHUN CHUN *, CLAES ESKILSSON†

* Underwood International College, Yonsei University, South Korea
email: sehun.chun@yonsei.ac.kr

† Department of Shipping and Marine Technology, Chalmers University of Technology, Sweden
email: claes.eskilsson@chalmers.se

A novel high-order numerical scheme is proposed to solve the shallow water equations (SWEs) on arbitrary rotating curved surfaces [1]. Based on the method of moving frames (MMF) [2] [3], the proposed scheme not only has the smallest dimensionality of two in space, but also does not require either of (i) metric tensors, (ii) composite meshes, or (iii) the surrounding space. The MMF-SWE formulation is numerically discretized using the discontinuous Galerkin method of arbitrary polynomial order p in space and an explicit Runge-Kutta scheme in time. In this talk, we start with the fundamental concepts of the innovational moving frames for Riemannian geometry developed by the famous French mathematician Elie Cartan in the early 20th century. Then, we discuss its adaptation and validity in the discrete space for scientific computing by overviewing the past works on conservational laws and diffusion equations. Applications to SWEs will be explained in details in views of algorithmic novelty to overcome the classical issues of PDEs on the closed surface such as geometric singularities and rotational effects. Results of six standard tests on the sphere will be displayed with the optimal order of convergence of $p+1$. Also, its general applicability and stability on arbitrary rotating surfaces such as ellipsoid, irregular, and non-convex surfaces will be demonstrated.

REFERENCES

- [1] S. Chun and C. Eskilsson, Method of moving frames to solve the shallow water equations on arbitrary rotating curved surfaces, *J. Comput. Phys.*, 333, 1-23, 2017
- [2] Sehun Chun, Method of moving frames to solve (an)isotropic diffusion equations on curved surfaces, *J. Sci. Compt.*, 59(3), 626-666, 2014
- [3] Sehun Chun, Method of moving frames to solve conservation laws on curved surfaces, *J. Sci. Compt.*, 53(2), 268-284, 2012

Towards a new class of Runge-Kutta HEVI time schemes for the fully compressible dynamical system

C. Colavolpe

The use non-hydrostatic fully compressible modelling system in the perspective of NWP and Climates applications raises many challenging questions, among which the choice time discretization scheme. It is commonly acknowledge that the integration of the fully compressible equations through the use of horizontally-implicit schemes creates scalability problems for massively parallel computing architectures. An alternative is to use horizontally explicit and vertically implicit (HEVI) approaches, where the implicit problems are only along the vertical direction. Besides, various multi-stage implicit-explicit (IMEX) methods, based on Runge- Kutta (RK) schemes, have been developed over recent years in order to achieve time- discretizations free of any computational modes, and possessing specified properties (accuracy- order,...). In this presentation, it is compared the analytical responses of three RK-IMEX HEVI schemes (identified as attractive for atmospheric modelling), for a linear fully compressible system supporting gravity and acoustic waves, as well as advection. Each scheme is analysed in two variants “UFPreF” and “UFPreB” recently proposed in the literature. The propagation of gravity waves is found to be generally be well represented, but the advection makes unstable all UFPreB variants, which were on contrary more stable without advection. The instability is analysed in a one-dimensional framework, and a new class of schemes is proposed to circumvent the problem using four Butcher tableaux, at no extra cost. Some numerical testing are provided to support the analyses in a more realistic context.

Recent improvements in the semi-Lagrangian transport in the spectral ECMWF model

M. Diamantakis, L. Magnusson, A. Agusti-Panareda,
W. Deconinck, S. Malardel, N.Wedi

Abstract

In March 2016 the horizontal resolution of the ECMWF Integrated Forecast System (IFS) was increased from 16km to approximately 9km. A key aspect of this upgrade was the introduction of the “octahedral cubic reduced Gaussian grid” [2] which resulted in improved efficiency, accuracy and mass conservation of the semi-Lagrangian, semi-implicit IFS. However, this upgrade exposed the problem of “noisy” vertical temperature structure in tropical cyclone forecasts. Numerical experimentation showed that this problem was a side-effect of insufficient convergence of the numerical algorithm used to compute the departure points (DP) in areas of high deformational Courant number (Lipschitz number) due to the use of very long timesteps [1].

In this talk we will review recent changes and work in the area of semi-Lagrangian transport in IFS prompted by the new high-resolution IFS. These changes resulted in improving further the efficiency and accuracy of IFS while they resolved the above mentioned tropical cyclone issues. In particular, standard algorithms for computing the DP based on fixed number of iterations will be compared versus a “dynamic iteration number” algorithm. Both approaches indicated that, for the long timesteps used by the IFS, the DP algorithm does not converge with the standard setting of two iterations and that it would be beneficial for the IFS accuracy to increase the number of these iterations [1]. Furthermore, the large improvements in model mass conservation will be briefly presented. This topic has two aspects: large reduction of the global total mass conservation error clearly attributed to the new cubic-reduced Gaussian grid and improvements in the transport of long-lived tracer gases such as CO₂, CH₄ in atmospheric composition forecasts using a modified version of the Bernejo & Conde mass fixer [3].

References

- [1] M. Diamantakis and L. Magnusson, *Sensitivity of the ECMWF model to semi-Lagrangian departure point iterations*, MWR 144 (2016), 3233-3250.
- [2] S.Malardel, N.Wedi, W.Deconinck, M. Diamantakis, C. Kühnlein, G. Mozdzynski, M. Hamrud, P. Smolarkiewicz, *A new grid for the IFS*, ECMWF newsletter No. 146 (2015)
- [3] A. Agusti-Panareda, M. Diamantakis, V. Bayona, F. Klappenbach, and A. Butz, *Improving the inter-hemispheric gradient of total column atmospheric CO₂ and CH₄ in simulations with the ECMWF semi-Lagrangian atmospheric global model*, GMD 10 (2016), 1-18

Dynamico-FE: A Structure-Preserving Hydrostatic Dynamical Core

Chris Eldred* (LAGA, University of Paris 13)
Thomas Dubos (LMD, Ecole Polytechnique)
Evangelos Kritsikis (LAGA, University of Paris 13)

*Presenting Author: chris.eldred@gmail.com

Abstract

Many important physical systems are Hamiltonian, and this structure underlies many of the most fundamental principles we know about nature (such as conservation laws). The equations of inviscid, (moist-)adiabatic fluid dynamics fall into this category, and form the basis of numerical models for climate and weather forecasting (termed dynamical cores). Guided by the philosophy that we are discretizing not arbitrary PDEs, but building representations of physical systems, it is highly desirable that a numerical model possess similar properties to the system under consideration. This can be achieved by retaining key elements of the underlying Hamiltonian structure in the discrete model. A very general approach to the design of such methods is outlined: the combination of a Hamiltonian formulation for the continuous equations with a mimetic discretization. Utilizing this approach, it is possible to obtain many desirable properties such as energy conservation, curl-free pressure gradients and the absence of spurious stationary computational modes. For several reasons (principally linear mode properties and higher-order accuracy on arbitrary grids), we have chosen to use a mimetic Galerkin method. Furthermore, by carefully choosing the discrete function spaces of this Galerkin method, it is possible to simultaneously obtain good computational performance on top of higher-order accuracy and the aforementioned desirable properties. This talk will present a concrete realization of this philosophy: Dynamico-FE, which is a high-order, structure-preserving hydrostatic dynamical core targeting moderate-resolution climate applications (around 25km in the horizontal). It will also discuss development efforts towards future sound-proof and non-hydrostatic dynamical cores built on the same principles, targeting high resolution, global convection permitting climate and weather forecasting.

Evaluating adaptive mesh refinement in 2D and 3D idealized atmosphere experiments

Jared Ferguson, Christiane Jablonowski, Hans Johansen,
Peter McCorquodale, Paul Ullrich, Phillip Colella

Adaptive Mesh Refinement (AMR) techniques have the potential to address the challenges of modeling tropical cyclones and other extreme weather and climate phenomena in a global GCM. By dynamically placing refined grids over salient transient features, models with AMR can provide sufficient local resolution while limiting the computational burden. This work explores the characteristics of AMR in a high-order finite-volume dynamical core on a cubed-sphere grid, in forced 2D shallow-water and idealized 3D dynamical core test cases. The shallow-water based test cases employ forcing mechanisms to mimic tropical cyclone-like vortex strengthening and orographically triggered features. They quantify the improvements gained from AMR grids, assess how well transient features are preserved across grid refinement boundaries, and determine criteria that maximize the AMR effectiveness. In addition, the corresponding non-hydrostatic 3D dynamical core will be introduced, including assessments via idealized 3D test cases that characterize the effect of AMR.

Hexagonal C-grid formulation of momentum diffusion and frictional heating

Almut Gassmann, IAP Kühlungsborn, Germany

Momentum diffusion extracts kinetic energy at the small scale end of the energy spectrum. In order to fulfill basic physical laws –such as energy conservation, angular momentum conservation and positive definite entropy production – this diffusion has to be formulated in terms of the Smagorinsky stress tensor. The stress tensor is constituted by strain and shear deformations.

The implementation of the momentum diffusion and the associated frictional heating turns out to be problematic on deformed hexagonal C-grid meshes. A straightforward discretization of the strain and shear deformations turns out to be not even convergent, if the hexagons are deformed. This is especially the case next to the 12 pentagon points on the sphere. Instead of formulating the diffusion in dependency of strain and shear deformations, a formulation with divergence and vorticity is presented. The variable spatial diffusion coefficient has then to be taken properly into account. The frictional heating cannot be evaluated by squared strain and shear deformations as it was the case for a regular mesh. An alternative method will be presented.

Note that for regular meshes, all the mentioned problems do not occur. The strain/shear deformation formulation and the vorticity/divergence formulation give the same results in this case.

As a side result of the investigation, it is again confirmed that the vorticity is not naturally defined via the Stokes theorem on triangles, but on a set of three rhombi. Furthermore, the usage of the vorticity components in the nonlinear generalized Coriolis term follows directly from one of the derivation steps.

In the experimental part it will be demonstrated that spurious diffusion near pentagon points present in the strain/shear deformation formulation is no longer present in the vorticity/divergence formulation of momentum diffusion.

Advances in the ACME-HOMME dynamical core

David Hall

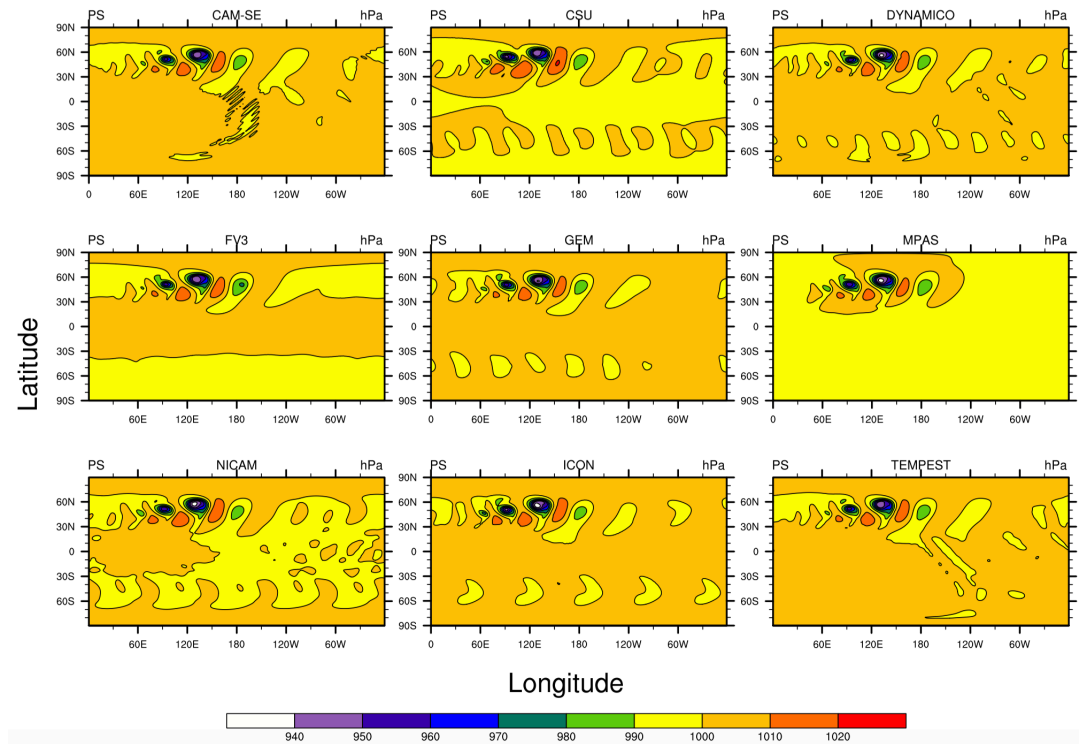
We will present an overview of recent advances made by our team in the development of the ACME version of the High-Order Methods Modeling Environment (HOMME) atmospheric dynamical core. Particularly we will discuss impacts of high-order vertical discretizations, new test cases, and progress on the nonhydrostatic version of the model.

DCMIP-2016: Overview and Results of the Moist Baroclinic Wave Test Case

Christiane Jablonowski (University of Michigan), Kevin A. Reed (Stony Brook University), Paul A. Ullrich (University of California, Davis), Colin M. Zarzycki (NCAR), James Kent (University of South Wales), Peter H. Lauritzen (NCAR), Ramachandran D. Nair (NCAR)

The 2016 Dynamical Core Model Intercomparison Project (DCMIP-2016) highlights the newest modeling techniques for global climate and weather models with particular focus on the newest non-hydrostatic global models, physics-dynamics coupling, and variable-resolution modeling. As part of a two-week summer school held in June 2016 at the National Center for Atmospheric Research (NCAR), a main objective of DCMIP-2016 was to establish an open-access database via the Earth System Grid Federation (ESGF) that hosts DCMIP-2016 simulations for community use from over 12 international modeling groups. The intercomparison is based on atmospheric model test cases of intermediate complexity that incorporate simplified physics parameterizations.

The talk presents the results of the first DCMIP-2016 test case which is an idealized moist baroclinic wave. The wave is triggered by a localized perturbation that overlays an analytically-prescribed moist reference state in gradient-wind and hydrostatic balance. The simple moisture feedbacks are represented by a warm-rain Kessler-type parameterization without any cloud stage. As the wave develops over 10 days (as e.g. shown by the surface pressure fields from nine DCMIP models at day 10 depicted below), this setup reveals the impact of the moisture processes on the development of the wave. The talk shows the characteristics of the test case and compares the results of the DCMIP-2016 models. This includes assessments of variable-resolution model configurations and the analysis of the coupling between the dynamics, physics and the tracer advection scheme. The latter is assessed as part of the baroclinic wave simulation via a “Terminator” tracer test that mimics photolysis-driven processes near the solar terminator. Overall, the work highlights that idealized test cases are part of a model hierarchy that characterizes and informs the design of atmospheric dynamical cores.



Abstract

Discontinuous Galerkin Discretization for Depth-averaged Non-hydrostatic Extension for Shallow Water Equations

Anja Jeschke (1,2), Stefan Vater (1,2) and Jörn Behrens (1,2)

(1) Department of Mathematics, Universität Hamburg, Hamburg, Germany

(2) Center for Earth System Research and Sustainability (CEN), Universität Hamburg, Germany

Numerical modeling of large-scale wave phenomena in ocean dynamics as tsunamis, storm surges and tides is often based on the shallow water equations. If the wave length becomes small or the wave propagates over rapidly changing bathymetry, the shallow water assumption loses its validity and a non-hydrostatic pressure component has to be considered. This allows the modeling of dispersive or 'non-hydrostatic' fluid flow meaning that waves of different wave lengths travel at different wave speeds. One approach to model dispersive waves is the depth-averaged non-hydrostatic extension for shallow water equations. We derive a conservative formulation of this approach and show its equivalence to some Boussinesq-type equations by assuming a quadratic vertical profile of the non-hydrostatic pressure instead of the traditionally linear one.

For the first time, a Runge-Kutta discontinuous Galerkin discretization is used for numerical modeling of the depth-averaged non-hydrostatic extension for shallow water equations. The numerical scheme is based on a projection method. In each timestep, a predictor step solves the hydrostatic shallow water equations. A correction step follows involving the solution of a first order elliptic system for the non-hydrostatic pressure. This system is also discretized using a discontinuous Galerkin discretization. Numerical tests validate our model concerning its dispersion relation and accurate representations of analytical solutions. We discuss the impact of the vertical profile of the non-hydrostatic pressure on our numerical results.

CAMR: An adaptive non-hydrostatic dynamical core for tracking atmospheric features

Hans Johansen, Jared Ferguson, Paul Ullrich,
Peter McCorquodale, Christiane Jablonowski

We present an adaptive mesh refinement (AMR) dynamical core for the cubed sphere, which is based on a finite volume discretization of the fully non-hydrostatic equations. The approach is conservative in scalar variables, and uses a new interpolation scheme that maintains 4th-order accuracy and continuity across the discontinuous metrics at cubed sphere panel boundaries. A horizontal explicit/vertical implicit (HEVI) approach splits the fast vertical acoustic waves from horizontal dynamics and allows for different time steps on each level of refinement, which greatly accelerates high-resolution global simulations of localized features. We present the dycore design, some validation results, and discuss some of the benefits (and difficulties) of doing AMR with HEVI schemes on cubed-sphere meshes.

A wavelet-based adaptive hydrostatic dynamical core

N.K.-R. Kevlahan¹, J. Feys¹ and T. Dubos² ¹*McMaster University, Hamilton, Canada, kevlahan@mcmaster.ca* ²*Ecole Polytechnique, France, dubos@lmd.polytechnique.fr*

At the previous two PDEs on the Sphere we presented a novel dynamically adaptive method for the shallow water equations on the sphere. This method is based on iterative dyadic refinement of the icosahedron, which generates a sequence of approximation subspaces (i.e. different grid levels) with arbitrarily fine local resolution controlled by a tolerance parameter. Biorthogonal wavelets are used to measure local error and to restrict or prolongate fluxes between different grid levels. The adaptivity is designed to preserve the mimetic properties (such as mass conservation) of the underlying discretization (in this case, TRiSK). A volume penalization technique is used to enforce no-slip boundary conditions at the coastlines when the model is used for ocean simulation. In this update we present a hydrostatic extension to three dimensions, based on the DYNAMICO model. The grid is adapted horizontally using the strictest condition over all vertical layers. We will discuss the results of a number of test cases from the DCMIP series. The goal of this project is to help assess the potential of dynamical adaptivity to improve the computational performance and numerical accuracy of climate models.

Split-explicit methods and local linear splitting

Oswald Knoth

Split-explicit methods are a common integration method in numerical weather prediction. They combine two explicit methods to integrate different parts of the right hand side with different time steps. Common combinations are for the slow part Leap-Frog, Runge-Kutta, or Adams-method and for the fast part a Verlet-type integration method. For Runge-Kutta methods as the slow integrator Wensch et.al give a generalization (MIS-method) and analyzed this new method in case of an exact integration of the fast part. An outcome of their analysis is that the methods of Wicker and Skamarock can have at least order two. In the talk we will apply this new methods to a splitting of the right hand side where the fast part is linear and is made of a partial Jacobian of the right hand side representing the fast acoustic waves. The method can be implemented by providing subroutines for computing the right hand side and a solver of choice for the linear differential equation. For special grid configurations like spherical grids or cut cell grids there are also stability restrictions for the slow waves due to small grid cells. In cooperation of this part of the slow operator in the linear part and applying special implicit-explicit methods for the integration of the linear part may be a possible extension of the split-explicit methods to this situation. We will compare our new integrators and known methods for the compressible Euler-equations on two dimensional cut-cell grids and spherical grids.

RCA5 - The New Rossby Centre Atmospheric Model

Marco Kupiainen Olof Grundestam Danijel Belusic David Lindstedt
Petter Lind

January 12, 2017

Abstract

The Rossby Centre at SMHI is developing a new solver for atmospheric simulations called RCA5. It is based upon the Flow-solver ESSENSE [1, 2]. The solver is a high-order finite difference method for the Navier-Stokes equations using Summation-By-Parts (SBP) operators augmented with weak boundary and interface conditions called Simultaneous Approximation Term (SAT) [3]. The theory and methodology for how to obtain approximate solution to partial differential equations with SBP-SAT is mature and well-documented [4]. We are now applying it to atmospheric simulations for the first time. We will present and motivate the development of the new code and also highlight the essential theory, where we will cover topics such as energy stability for semi-discrete approximations. We will also show results from standard atmospheric test cases, e.g. from DCMIP [5].

References

- [1] Magnus Svärd, Mark H. Carpenter, and Jan Nordström. A stable high-order finite difference scheme for the compressible Navier Stokes equations, far-field boundary conditions. *Journal of computational physics*, 225:1020–1038, 2007.
- [2] Magnus Svärd and Jan Nordström. A stable high-order finite difference scheme for the compressible navier-stokes equations: Wall boundary conditions. *Journal of Computational Physics*, 2008.
- [3] M. H. Carpenter, J. Nordström, and D. Gottlieb. A stable and conservative interface treatment of arbitrary spatial accuracy. *Journal of Computational Physics*, 1999.
- [4] M. Svärd and J. Nordström. Review of summation-by-parts schemes for initial-boundary-value problems. *Journal of Computational Physics*, 2014.
- [5] P. A. Ullrich, C. Jablonowski, J. Kent, P. H. Lauritzen, R. D. Nair, and M. A. Taylor. Dynamical core model intercomparison project (dcmip), test case document. https://earthsystemcog.org/site_media/docs/DCMIP-TestCaseDocument_v1.7.pdf.

On the development of CAM-SE-CSLAM with separate physics grid

Peter Lauritzen

The new NCAR version of CAM-SE-CSLAM (Community Atmosphere Model using the Spectral Elements dynamical core and accelerated transport with Conservative Semi-Lagrangian Multi-tracer scheme) involves several science, software engineering and performance upgrades. This model version uses a dry-mass vertical coordinate, condensate loading, has the ability to run physics on a different grid than dynamics and provides consistent finite-volume transport of tracers (CSLAM) that improves the simulation in terms of accuracy and throughput. We also provide “out-of-the-box” capability to run in simplified settings (for example, moist baroclinic wave, ‘toy’ terminator chemistry and simple physics).

This talk will mainly focus on the development of a separate physics grid in CAM-SE-CSLAM when using the comprehensive CAM physics package. Conservation and the sensitivity to the accuracy of the mapping algorithm will be discussed.

**A Characteristic Discontinuous Galerkin Method for Tracer Advection in
MPAS-Ocean
FEF 2017**

David Lee*

* Los Alamos National Laboratory
Los Alamos, New Mexico, USA
e-mail: drlee@lanl.gov

ABSTRACT

A new characteristic discontinuous Galerkin (CDG) advection scheme is presented. In contrast to standard discontinuous Galerkin schemes, the test functions themselves follow characteristics in order to ensure conservation and the edges of each element are also traced backwards along characteristics in order to create a swept region, which is integrated in order to determine the mass flux across the edge. Similar to semi-Lagrangian schemes, the accuracy and performance of the scheme are improved by the use of large CFL numbers and the scheme scales sublinearly with the number of tracers being advected. Using a modal Taylor series basis, the CDG scheme may be run to arbitrarily high order spatial accuracy and on unstructured grids. The scheme is being developed for implementation within the Model for Prediction Across Scales (MPAS) Ocean model, where it is applied on an unstructured Voronoi grid in the horizontal and a temporally evolving arbitrary Eulerian-Lagrangian grid in the vertical by way of operator splitting.

REFERENCES

[1] D. Lee, R. Lowrie, M. Petersen, T. Ringler, M. Hecht “A high order characteristic discontinuous Galerkin scheme for advection on unstructured meshes”, *J. Comp. Phys.* Vol. **324**, pp. 289–302, (2016).

Impacts of dynamical solvers on medium-range weather forecasts

Shian-Jiann Lin, NOAA/Geophysical Fluid Dynamics laboratory

January 13, 2017

The numerical and software design of the Finite-Volume dynamical core on the cubed-sphere (FV3) will be presented, with focus on FV3's key scientific and computational advantages, such as the vertically Lagrangian discretization, high-order finite-volume advection of potential temperature, air and tracer mass, and perhaps more importantly, consistent advection of absolute vorticity with afore-mentioned scalars. It is noted that FV3's C-D grid staggering of the horizontal winds with high-order finite-volume discretization, which was designed to achieve the goal of consistent PV transport, was difficult, if not impossible, to analyze by standard linear analyses, which led to some misleading results by some authors.

For the next Generation Global Prediction System (NGGPS), the non-hydrostatic FV3 enables the unification of regional and global models, for both weather and climate applications. This is made possible by choosing the scalable cubed-sphere grid that can easily accommodate additional regional "tiles" for two-way nesting purpose. A grid stretching capability is also built into FV3 to enable a smooth variation of resolution within the global domain. As such, FV3's computational efficiency is significantly higher than traditional variable-resolution or the Adaptive Mesh Refinement (AMR) approaches.

The FV3 has been used in production at the Geophysical Fluid Dynamics Laboratory (GFDL) and at NASA for climate simulations during the past decade. With the NGGPS project (phase-1 and phase-2), FV3 has also been evaluated in Numerical Weather Prediction (NWP) mode. Impacts of the non-hydrostatic FV3 dynamical core to medium-range weather predictions will be presented using interpolated analysis fields from both NCEP and ECMWF, with resolution at 13-km as well as a "global cloud-permitting" 3-km configuration.

Improving Balancing Properties in the Terrain-Following Hydrostatic Regional Climate Model REMO

L. Marien¹, S. Vater², J. Behrens^{2,3}, and D. Jacob¹

¹Climate Service Center Germany (GERICS), Fischertwiete 1, D-20095 Hamburg, Germany

²CEN—Center for Earth System Research and Sustainability, University of Hamburg, Grindelberg 5, D-20146 Hamburg, Germany

³Department of Mathematics, University of Hamburg, Bundesstraße 55, D-20146 Hamburg, Germany

Abstract

Accurate projection of local precipitation patterns is a long standing problem in climate modeling. Among numerous other issues related to physical parameterization schemes and quality of available observations, prior research has identified erroneous representation of the pressure gradient force (PGF) as a possible compounding factor especially in mountainous regions. In such cases the source of error in the PGF is the use of terrain-following coordinates which are ubiquitous in climate modeling. More specifically in these coordinate systems the PGF is often represented as the sum of two terms about equal in magnitude but with opposite signs, giving rise to unbalanced truncation errors in standard numerical discretizations. In this work we investigate this problem of well-balancing in the context of the finite difference hydrostatic regional climate model REMO. We propose an alternative discretization of the PGF based on local and time-dependent hydrostatic background states constructed by analytically solving appropriate initial-value problems for the hydrostatic equation. For the simplified case of a computational grid with co-located pressure and temperature variables this approach yields substantial improvements in idealized test cases. For practical applications however a generalization to the staggered Lorenz-grid employed in REMO has to be derived. In this context we find that particular challenges arise from pressure based coordinates as used in REMO in comparison to those based on height. We investigate different generalization strategies and demonstrate their individual merits and shortcomings based on the results of several two-dimensional test cases.

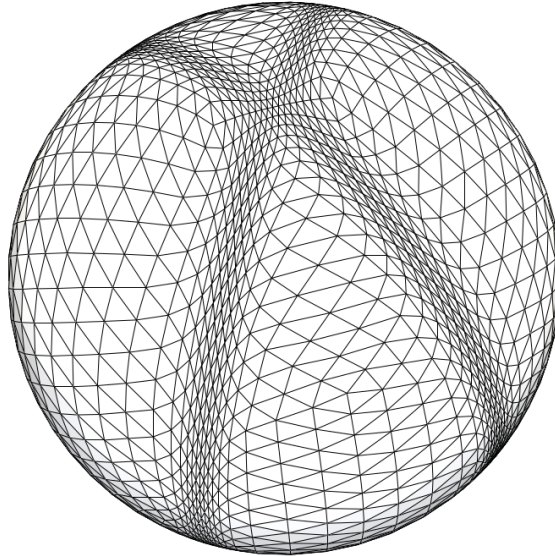
MESH ADAPTIVITY FOR NWP USING OPTIMAL-TRANSPORT-BASED METHODS

Andrew T. T. McRae^{*,1}, Colin J. Cotter², Jemma Shipton² and Chris J. Budd¹

¹ Department of Mathematical Sciences, University of Bath

² Department of Mathematics, Imperial College London

We discuss the generation of meshes adapted to a prescribed scalar monitor function, in the context of a moving mesh method for NWP. This is done through equidistribution, so that the volume of a cell is inversely proportional to the monitor function. Together with an optimal transport condition, this requires (on the plane) the solution of a nonlinear equation of Monge-Ampère type, as performed in Budd and Williams (SISC, 2009). As our ultimate interest is in numerical methods for global NWP, we formulate and solve the equivalent equation for generating analogous meshes on the sphere. We present robust iterative numerical methods for doing so, using finite element methods to solve the nonlinear equations. We finish by discussing the extension of the compatible finite element method approach of Cotter and Shipton (JCP, 2012) to solving 2D geophysical equations posed on these dynamically adaptive meshes.



LFRic: Scalability and flexibility of models on future HPCs

Met Office, STFC, University of Manchester

The joint Met Office, NERC, and STFC GungHo project proposed a new dynamical core for the Met Office’s weather and climate models that targets efficient use of next generation High Performance Computers (HPCs). That new dynamical core is based on a finite-element discretization applied to the cubed-sphere. The current Unified Model software infrastructure is unsuitable for such a change to the model and is unlikely to meet the challenges of the next generation of supercomputers. Therefore the GungHo project additionally proposed a new modelling infrastructure. The LFRic project (named after L.F. Richardson) is implementing that design and will replace the Met Office’s Unified Model. As well as having to support the finite element data structures required by GungHo, LFRic must be flexible to changes in HPC architecture or model formulation (and hence is based on unstructured meshes), and deliver good computing performance on future exascale supercomputers.

To achieve these goals, the scientific code has been separated from the underlying computational infrastructure, so that changes can be made to either without impacting the other. Code auto-generation techniques are used to optimise the model for different system architectures. This split of responsibility is termed the “Separation of Concerns”. Scaling experiments on the new Met Office supercomputer using auto-generated code confirm the viability of the approach.

Next steps of development involve the infrastructure to support the science requirements (multigrid solver, physics-dynamics coupling, kernel optimisation), work on scalable input-output for diagnostic and ancillary data, and building and testing climate and weather configurations, with the aim of migrating Met Office systems from ENDGame to LFRic once science and performance targets are met.

FOURTH-ORDER TIME-STEPPING FOR STIFF PDES ON THE SPHERE**HADRIEN MONTANELLI***, **YUJI NAKASTUKASA†***Oxford University Mathematical Institute, Oxford OX2 6GG, UK.
email: montanelli@maths.ox.ac.uk†Oxford University Mathematical Institute, Oxford OX2 6GG, UK.
email: nakatsukasa@maths.ox.ac.uk

We present in this talk algorithms for solving stiff PDEs on the unit sphere with spectral accuracy in space and fourth-order accuracy in time. These are based on a variant of the double Fourier sphere method in coefficient space and implicit-explicit time-stepping schemes. The use of multiplication matrices operating on the Fourier coefficients avoids the artificial pole singularity while implicit-explicit schemes circumvent severe restrictions on the time-steps due to the clustering of points near the poles. A comparison is made against exponential integrators and it is found that implicit-explicit schemes perform best. Implementations in MATLAB and Chebfun make it possible to compute the solution of many PDEs to high accuracy in a very convenient fashion.

ESCAPE: Optimising NWP Dwarfs for Energy Efficient Exascale Computing

Andreas Mueller, Willem Deconinck, Peter Bauer, Nils Wedi
Earth System Modelling Section
ECMWF, Shinfield Park, Reading RG2 9AX (UK)

Gianmarco Mengaldo
Division of Engineering and Applied Sciences
California Institute of Technology, CA 91125 (USA)

In the simulation of complex multi-scale flow problems, such as those arising in weather and climate modelling or in engineering, one of the biggest challenges is to satisfy operational requirements in terms of time-to-solution and available energy without compromising the accuracy and stability of the solution. These two competing factors require extreme computational capabilities in conjunction with state-of-the-art algorithms that can optimally suit the targeted underlying hardware while improving the convergence to the desired solution. In this picture, it is also necessary to consider the additional complexity cast by the rapidly evolving and more energy-efficient computing hardware to which simulation technologies must adapt.

The European Centre for Medium Range Weather Forecasts (ECMWF) is leading the H2020 FET-HPC project ESCAPE of innovation actions for developing a holistic understanding of energy-efficiency for extreme-scale applications using heterogeneous architectures, accelerators and special compute units by (a) testing different numerical techniques to approximate the dynamical core of the global Numerical Weather Prediction (NWP) model; (b) combining frontier research on algorithm development for use in extreme-scale, high-performance computing applications, minimising time-and energy-cost-to-solution; and (c) synthesising the complementary skills of global NWP with leading European regional forecasting consortia, university research, experienced high-performance computing centres and hardware vendors.

In addition to introducing the ESCAPE project this talk will present performance analysis for the spectral transform and MPDATA advection dwarf as well as first steps towards improving their performance.

Acknowledgements: The ESCAPE project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 671627.

A Split-Explicit Time Integration Scheme for the Godunov-Type Nonhydrostatic Finite-Volume Model.

Ram D. Nair and Farshid Nazari

Institute for Mathematics Applied to Geosciences
National Center for Atmospheric Research (NCAR)
1850 Table Mesa Drive, Boulder, CO 80305, USA.
Email: rnair@ucar.edu

A large class of finite-volume (FV) methods for solving hyperbolic conservation laws are based on high-order extensions of the classical Godunov scheme, often referred to as the Godunov-type schemes. They offer computationally attractive features including inherent conservation, geometric flexibility, accuracy and high parallel efficiency. The Godunov-type method typically does not rely on staggered grids, and the cell-averaged solution is not assumed to be continuous across the cell (control volume) edges. The discontinuity of the fluxes at the cell interface is resolved by a Riemann solver (numerical flux). We consider an upwind-based Godunov-type FV method for solving nonhydrostatic (NH) atmospheric flows (fully compressible Euler system of equations) on a rectangular 2D (x, z) -domain. The fluxes at the cell interface are reconstructed by the fifth-order accurate Piecewise Quartic Method (PQM), in a dimension-split manner. The AUSM⁺-up numerical flux is used for the FV model, which is particularly effective for low Mach number problems such as the NH atmospheric modeling.

A major challenge for NH modeling is to develop a practical time-stepping method. This is because of the wider spectrum of spatial and temporal scales present in the atmosphere, encompassing fast sound and gravity wave propagation as well as the slower advection. Although the explicit Runge-Kutta (RK) scheme is robust and accurate, it is not an efficient choice for NH modeling because the stringent CFL stability restriction associated with a combination of acoustic waves and small grid spacing. A practical strategy to take the advantages of parallel-efficient explicit schemes is a splitting approach, in which the equations of motion are split into fast-slow components. The time-step size is then restricted by the CFL number of the low-frequency modes, since smaller time steps are applied for the integration of the high-frequency modes. However for a Godunov-type FV method, it is not obvious to separate each component of the Euler system and perform regular split-explicit time integration. We propose a three-mode splitting (fast, medium, and slow) approach based on the scale analysis in the atmosphere, and perform a multi-rate time integration. The computational efficiency of the split-scheme is compared with the explicit RK scheme using the FV model, with various NH benchmark test cases.

Implementation of the Spectral Element Lagrangian Transport (SPELT) Scheme in the Non-hydrostatic Global Atmospheric Model of KIAPS

Hyun Nam and Suk-Jin Choi

Korea Institute of Atmospheric Prediction Systems(KIAPS), Korea

E-mail: h.nam@kiaps.org and sj.choi@kiaps.org

Abstract

With the purpose of enhancing performance of tracer advection for positive-definite and shape-preserving properties, the conservative semi-Lagrangian scheme (SPELT - Spectral Element Lagrangian Transport) (Erath and Nair, 2014) is implemented in the non-hydrostatic global atmospheric model (Choi and Hong, 2016). The model runs with the current spectral element tracer transport scheme on fixed grids of Eulerian type in the explicit third order Runge-Kutta time-advancing. On non-uniform quadrilateral grids (in 2D), SPELT scheme is mass conservative, multi-tracer efficient, third order accurate and scalable on parallel computers. Also, the scheme avoids complex and expensive upstream area computations. From the numerical results, we compare SPELT scheme (semi-Lagrangian type) with the spectral element tracer transport scheme (Eulerian type) in the non-hydrostatic global atmospheric model. It is shown to confirm the efficiency and reliability of the proposed scheme.

Narrowing Constraints: A Wide View on Algorithms and Science Goals for Climate and Weather in Light of Modern Architectures

Matthew R. Norman

January 17, 2017

"There is no free lunch." This saying all-too-accurately describes the difficulty of making substantive advances in weather and climate applications in terms of algorithms and scientific approaches. The reality is that our communities have ridden certain reliable waves for quite while, and those waves appear to have reached their breaking points (if not already broken). There are a number of constraints for modern, accelerated, parallel computing that are becoming increasingly narrow. Cores are being slowed down, data bandwidth increases require increasingly large blocks of *contiguous* accesses, data latencies are not improving at all, off-node data transfer is choking our model throughput, time-explicit time step reduction is stretching the work per node too thin, and global data movement inherent to pre-conditioned implicit solutions is increasingly costly. One thing is clear: the days of "business as usual" are over, at least for climate.

This presentation discusses these narrowing constraints and their interaction with both the large-scale algorithmic approaches and smaller-scale algorithmic choices we make in our models. Finally, some of the novel "front runners" will be presented that seem to be addressing these narrowing constraints in a direct and positive manner.

A theory of Kelvin and Yanai waves on a sphere derived from approximate Schrödinger equations

Nathan Paldor

Linear wave theory of the Rotating Shallow Water Equations (RSWE, hereafter) yields a simple and concise scenario of westward propagating waves in the unbounded equatorial beta-plane but a very complicated scenario on a sphere. In particular, currently no theory is available for Kelvin and Yanai waves on a sphere though these waves could have been employed for the construction of test cases for global scale dynamical cores. In my talk I will combine exact numerical (i.e. differential shooting) solutions of the RSWE with analytic solutions of associated, approximate, Schrodinger eigenvalue equations to calculate the dispersion relations and meridional amplitude structure of these waves on a sphere. Different Schrodinger eigenvalue equations approximate the RSWE in different ranges of : Lamb number, frequency and zonal wavenumbers. Kelvin wave which is derived on the equatorial beta-plane by simply letting the meridional velocity component vanish does not exist as a separate additional mode on a sphere. Instead, the eastward propagating $n=0$ Inertia-Gravity (IG) mode is nearly non-dispersive and, as in all IG modes, the corresponding meridional velocity is small so this $n=0$ IG mode can be named a “Kelvin” wave. I will also show that Yanai wave (AKA the “mixed mode”) exists on a sphere as the $n=0$ westward propagating mode but, unlike the equatorial beta-plane, a second westward propagating (the anti-Kelvin) does not. The reasons for the absence of a second westward propagating mode on a sphere are different from those on the equatorial beta-plane. Specifically, on the equatorial beta-plane the absent second mode is associated with singular zonal velocity while on a sphere the zonal velocity is regular at all wavenumbers and mode numbers. In addition, the dispersion curves of low modes of the westward propagating Inertia-Gravity waves have zonal wavenumbers in which the group velocity vanishes. This finding might be of importance in determining the dominant zonal wavenumbers in the atmosphere.

Extension of the vertical discretization with the finite element method to the non-hydrostatic dynamical core of KIAPS

Ja-Rin Park and Suk-Jin Choi

Korea Institute of Atmospheric Prediction Systems, Seoul, Republic of Korea

Abstract

This is the preliminary study of the vertical finite element method (VFE) to non-hydrostatic dynamical core developed in KIAPS^[1]. VFE resulted in the high accuracy for the hydrostatic primitive equations system^[2]. It is naturally extended to the non-hydrostatic (NH) dynamical core which solves the fully compressible flux-form Euler equations using perturbation variables with the spectral element and finite difference methods of horizontal and vertical direction, respectively. VFE's integral and derivative operators are defined with basis functions using the linear and cubic splines on no staggering grids.

Since the prognostic variables of the model is coupled with mass (i.e. flux-form), it is revealed that the invertible property between the integral and derivative operations is important. The invertibility of finite element based operators has potentials to cause the instability of the system so that some calculations of the vertical velocity are manipulated to avoid sequential application of the vertical integral and derivative operators during time integration. In this presentation, the behavior of the VFE in NH system will be discussed using several idealized test cases.

Reference

- [1] Choi, S.-J. and S.-Y. Hong, 2016: A global non-hydrostatic dynamical core using the spectral element method on a cubed-sphere grid, *Asia-Pac. J. Atmos. Sci.*, 52(3), 291-307.
- [2] Yi, T.-H. and J.-R. Park, 2016: Vertical Discretization with Finite Elements on the Cubed Sphere for a Global Hydrostatic Model, Submitted to *Journal of Computational Physics*, in revision.

Numerical instabilities of vector invariant momentum equations on C-grids

Michael J. Bell (Met Office, UK)
Pedro S. Peixoto* (Universidade de São Paulo, Brazil)
John Thuburn (University of Exeter, UK)

*Presenting Author: `pedrosp@ime.usp.br`

Abstract

Hollingsworth et al. (1983) discovered that certain implementations of energy and enstrophy conserving schemes for vector invariant hydrostatic primitive equations models were prone to near-grid-scale instabilities, with severe consequences to high resolution forecasts. There has been a renewed interest in these instabilities, as they are present in modern C-grid based atmospheric and ocean models.

This talk discusses the recent results of Bell et al. (2017), which analyses the problem decomposing the numerical modes into horizontal and vertical components, showing, among other things, that it is possible to capture the instability within shallow water models context. A key point is that the instability is more prominent under small equivalent depths. With this in focus, we will also discuss new results about the instability on spherical shallow water models that are built on unstructured C-grids.

References

- [1] Hollingsworth, A., Kllberg, P., Renner, V. and Burridge, D. M. (1983), An internal symmetric computational instability. *Q.J.R. Meteorol. Soc.*, 109: 417428.
- [2] Bell, M. J., Peixoto, P. S. and Thuburn, J. (2017), Numerical instabilities of vector-invariant momentum equations on rectangular C-grids. *Q.J.R. Meteorol. Soc.*

Distributed ADI preconditioning of elliptic solvers in all-scale global models of atmospheric flows

Zbigniew Piotrowski, Piotr Smolarkiewicz
IMGW, PSNC, ECMWF

Effective operator preconditioning lies at the heart of multiscale flow simulation, including a broad range of geoscientific applications that rely on semi-implicit integrations of the governing PDEs. For such problems, conditioning of the resulting sparse linear operator directly responds to the squared ratio of largest and smallest spatial scales represented in the model. For thin-spherical-shell geometry of the Earth atmosphere the condition number is enormous, upon which implicit preconditioning is imperative to eliminate the stiffness resulting from relatively fine vertical resolution. Furthermore, the anisotropy due to the meridians convergence in regular latitude-longitude discretizations becomes equally detrimental as the horizontal resolution increases to capture nonhydrostatic dynamics. Herein, we discuss a class of effective preconditioners based on the parallel ADI approach. The approach has been implemented in the established high-performance all-scale model EULAG with flexible processor distribution including a 3D array. EULAG offers both : Eulerian and Semi-Lagrangian advection, the latter enabling long-timestep integrations on anisotropic grids. The efficacy of the approach is demonstrated in the context of an archetypal simulation of global weather.

Monotonicity and mass conservation for tracer transport in GEM model

Abdessamad Qaddouri, Monique Tanguay, Jean de Grandpré

The global environmental multiscale (GEM) model uses the standard semi-Lagrangian (SL) transport scheme for the solution of the advection problem on the Yin-Yang grid system. The magnitude of the impact of the lack of formal mass conservation in the SL scheme due to interpolation step depends on the nature of the advected quantity and particularly on the strength of species spatio-temporal variability. Different mass fixer algorithms (MFAs) and shape preserving schemes (SPSs) have been implemented in the GEM model to ensure monotonicity and mass conservation of atmospheric tracers. In this talk we present a methodology based on combining different established algorithms in order to achieve monotonicity and mass conservation with transport SL scheme on the overset grid Yin-Yang. The method must not degrade substantially the order of the overall scheme despite the extra constraints on the monotonicity and conservation. We will also present some preliminary results from our implementation of SL Inherently Conserving and efficient scheme (SLICE) in the GEM model. Numerical tests include passive advection with idealized flows as well as the advection of non-passive tracers such as ozone.

A multiscale coupled atmosphere-ocean oscillator

Enver Ramirez

In the present work, a simplified multi-space and multi-time atmosphere-ocean coupled oscillator for the interactions between the synoptic-intraseasonal-interannual scales is developed. The oscillator is made of two coupled nonlinear equatorial beta-plane shallow water equations, one representing the ocean and the other the atmosphere. Different from previous studies, linear like wave modes are allowed in both subsystems and two types of nonlinearities are included, i.e., the intrinsic advective nonlinearity and the atmosphere-ocean coupling related. Simplified physical parameterizations for the air-sea coupling are developed. To mimic the main differences between the fast-atmosphere and the slow-ocean, suitable multi-space and multi-time scalings are applied yielding a balanced synoptic/Intraseasonal/Interannual-El Niño (SInEN) regime. In the model, the synoptic scale is the fastest atmospheric scale, the intraseasonal is the intermediate air-sea coupling scale and the El Niño refers to the slowest interannual ocean scale. The analytical solutions of the SInEN equations for a discrete resonant triad can be expressed in terms of Jacobi Elliptic functions, the analytical solutions highlights that the slow wave amplitude evolution depends on both types of nonlinearities and on the initial energy partition among the triad members. The energy is able to flow between the triad members in a much more slower period than those of the involved mode members. The oscillation period depending on the amplitude of the oscillation is also found in large amplitude oscillator. Physically the energy exchanges occurs through the following processes : Nonlinear interactions of synoptic scale atmospheric waves force intraseasonal variability in the atmosphere but also in the ocean through wind stress. Intraseasonal ocean temperature perturbations coupled with the atmosphere through evaporation force higher order atmospheric variability and the wave-convection coupling provides another source of higher order atmospheric variability. In the ocean, nonlinear interactions of intraseasonal ocean perturbations force interannual oceanic variability. The slowest interannual variability in the SInEN regime is associated with either nonlinear wind stress and advective nonlinearity. The SInEN oscillator illustrates a way to connect the atmosphere and ocean from synoptic to interannual time-scales through intraseasonal time-scale.

DCMIP-2016: Results of the Tropical Cyclone and Supercell Test Cases

Kevin Reed, Colin Zarzycki, Christiane Jablonowski, Paul Ullrich,
James Kent, Peter Lauritzen, Ramachandran Nair

The 2016 Dynamical Core Model Intercomparison Project (DCMIP-2016) held at the National Center for Atmospheric Research (NCAR) in June 2016 utilized test cases of intermediate complexity to compare and explore advances in non-hydrostatic global models. In particular, DCMIP-2016 paired 12 different dynamical cores with a simplified physical parameterization package to shed light on the physics-dynamics interactions in a controlled test environment.

The talk presents the results of an idealized tropical cyclone and supercell test cases, which are the second and third of the three DCMIP-2016 tests. The tropical cyclone test is based on an analytically-prescribed vortex in a quiescent background environment that fosters a rapid intensification of the initial tropical cyclone seed. The cyclone intensifies from a weak, balanced vortex to a tropical cyclone over a 10-day period. Moisture and physics-dynamics coupling features prominently in this test, as well as the impact of precipitation and boundary layer parameterization formulations. The DCMIP-2016 models are compared at a default horizontal grid spacing of approximately 50 km, and effects of higher horizontal resolutions are explored. The supercell storm test permits the study of a non-hydrostatic moist flow field with strong vertical velocities and associated precipitation. This test assesses the behavior of global modeling systems at extremely high spatial resolution and is used in the development of next-generation numerical weather prediction capabilities. In this regime the effective grid spacing is very similar to the horizontal scale of convective plumes, emphasizing resolved non-hydrostatic dynamics. The work demonstrates how idealized test cases are part of a model hierarchy that helps distinguish between causes and effects in atmospheric models and the physics- dynamics interplay.

18th PDEs on the Sphere Workshop
3-7 April, 2017 Paris, France

NEXT GENERATION NWP USING A SPECTRAL ELEMENT DYNAMICAL CORE

ALEX REINECKE¹, KEVIN VINER¹, SASA GABERSEK¹, MATUS MARTINI², JAMES DOYLE¹, JOHN MICHALAKES³, DAVID FLAGG¹, AND DAVID RYGLICKI³

¹Naval Research Laboratory, Monterey, CA, USA.

²Devine Consulting, Monterey, CA, USA.

³ University Corporation for Atmospheric Research, Boulder, CO, USA.

⁴ National Research Council, Monterey, CA, USA.

The Navy Environmental Prediction System Using the NUMA Core (NEPTUNE) is a next generation global cloud resolving numerical weather prediction (NWP) system being developed at the Naval Research Laboratory (NRL) and the Naval Postgraduate School (NPS). The dynamical core of NEPTUNE is based upon a three-dimensional spectral element numerical representation of the continuous non-hydrostatic, deep-atmosphere Euler equations. In this talk we will discuss the implementation and testing of NEPTUNE using real-data initial conditions and the performance of different physics packages in the spectral element framework. We will present real-data results comparing NEPTUNE to the existing Navy global model, NAVGEM. The computational attributes of the NEPTUNE model will be discussed. Examples will be presented from both a global and limited area configurations of NEPTUNE highlighting real data and idealized model evaluations.

Development of a Hexahedral Yin-Yang Grid Global Model: AGHEXA

M. Sakamoto, K. Aranami, K. Kawano, K. Matsubayashi, T. Hara, H. Kurahashi, J. Ishida,
Japan Meteorological Agency, Tokyo, Japan
Y. Kitamura

Meteorological Research Institute, Tsukuba, Ibaraki, Japan

1. Introduction

Yin-Yang grid is a composite mesh proposed by Kageyama and Sato (2004), and two types of spherical curvilinear coordinates, in which positions of poles are different, are combined to cover the whole spherical domain. Several types of Yin-Yang grids have been proposed in the past studies (Qaddouri and Lee 2011, Baba et al. 2010, Peng et al. 2006).

A hexahedral grid shown in Figure 1 was proposed by Sakamoto et al. (2015) referring to Dudhia and Bresch (2002). This design reduces the overlap areas (surplus calculation domains), and is useful to suppress negative effects arising from the overlap. The six calculation domains have similar size, and this is convenient when considering the parallel computing strategies. AGHEXA is a composite mesh global model which uses this hexahedral mesh and a finite volume method dynamical core ASUCA (Ishida et al. 2010).

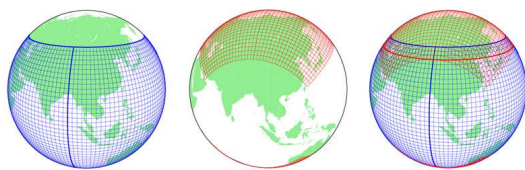


Figure1. The hexahedral Yin-Yang mesh used for AGHEXA. Blue domains are the Yin (ordinal lat-lon) coordinate systems, and red domains are the Yang coordinate systems. Thick blue and red lines are the edges of the calculation domains. The areas in between the thick lines are the overlapped areas where two predictions are executed.

2. Results of Idealized Test Cases

Idealized test cases for global models were examined as part of the studies by the high performance computing infrastructure projects in Japan. Results of AGHEXA are shown.

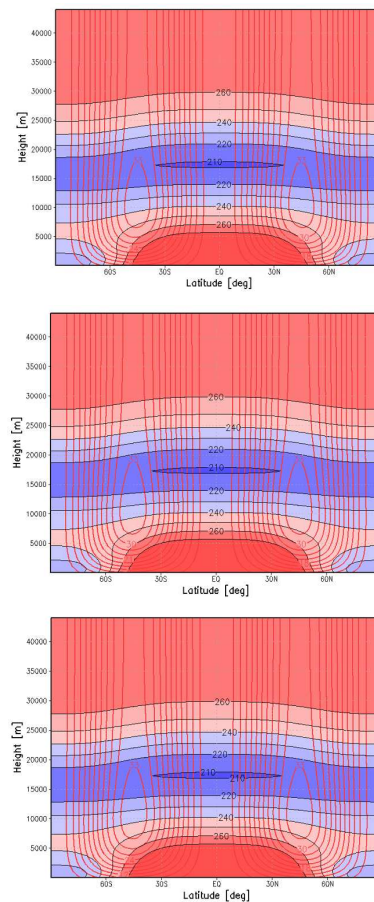


Figure2. Result of the steady-state test using the iso-geopotential lower boundary. The upper panel shows the initial state, middle 5day, lower 10day forecast results. Shades: temperature (K), red lines: wind speed (contour interval 3m/s).

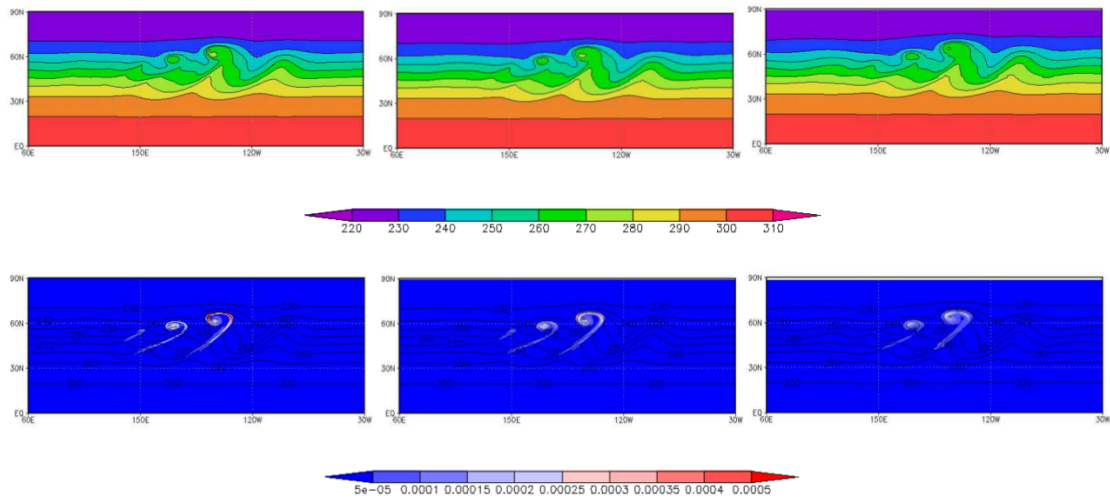


Figure 3. Results of the baroclinic-wave test using the iso-geopotential lower boundary. Nine day forecasts of temperature (K) [upper panels] and relative vorticity (1/s) [lower panels] at 850hPa are shown. Three panels in each row were by a 0.25deg resolution [left], 0.5deg [center], and 1.0deg [right] respectively.

2-a) Jablonowski and Williamson (2006)

The iso-geopotential lower boundary was used according to a suggestion by Dr. Yoshida of the RIKEN Advanced Institute for Computational Science (AICS), who presented similar test results for NICAM, ICON, MPAS, and DYNAMICO. Adopting the iso-geopotential lower boundary is convenient for comparing with his results.

Figure 2 shows results for the steady-state test case. AGHEXA shows reasonable stability for 10 days, which is similar to the results by the other models. Figure 3 shows nine day forecasts of temperature and vorticity at 850 hPa for the baroclinic-wave test. As the horizontal resolution increases, the shape of the vorticity disturbance becomes sharp and intense. Results will be detailed at the PDEs workshop.

2-b) Klemp, Skamarock, and Park (2015)

We have been examining the supercell simulation test on the reduced-radius sphere.

Figure 4 shows the results for the 0.5 degree grid test. The numerical diffusion coefficient used is $20 \text{ m}^2/\text{s}$ instead of $500 \text{ m}^2/\text{s}$ because the value specified in Klemp, Skamarock, and

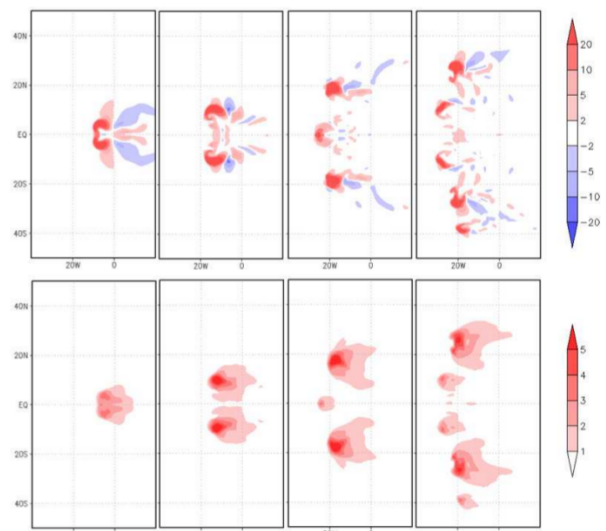


Figure4. Vertical wind speed (m/s) [upper panels] and rain water (g/kg) [lower panels] at 5,250m above the surface of the supercell test. Panels in each row are 30 min [left end], 60 min [second from left], 90 min [third], and 120 min [right end] from the initial state. As for the numerical diffusion, $\nu=20\text{m}^2/\text{s}$ is used instead of $500\text{m}^2/\text{s}$.

Park (2015) is too large to conserve the strong convection arising from the perturbation for two hours. We are still investigating the results including the dependency on the horizontal resolution, and will discuss at the PDEs workshop.

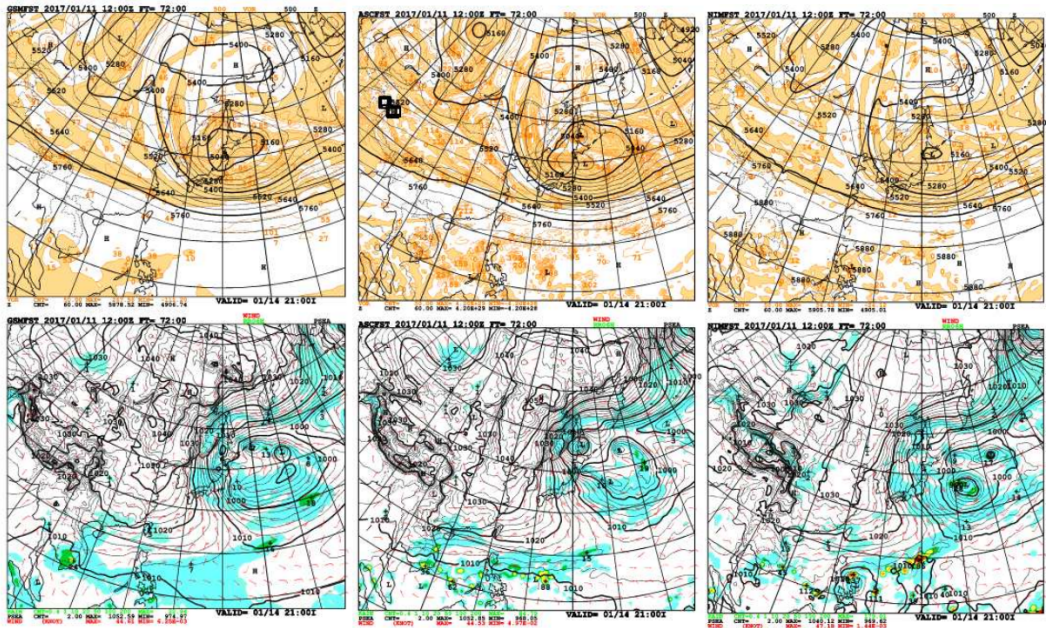


Figure 5. An example of dairy forecast tests. 72 hour forecasts for geopotential height (m) and relative vorticity (10^{-6} /s) at 500hPa [upper panels], and pressure (hPa) at the mean sea level and 6 hour precipitation (mm) [lower panels]. Left panels are by JMA's operational global forecasting, the center ones by AGHEXA (0.125deg grid, 60layers, without any convective parameterization), and the left ones by NICAM (glevel-9, 40layers with a convective parameterization). Solid thick lines over the Tibet Plateau in AGHEXA's upper panel are areas where surface pressure is below 500hPa.

3. Realistic weather forecast tests.

84 hour predictions by AGHEXA (0.125deg grid, 60 layers up to about 45km, 40 sec time step) have been executed and examined on HITACHI SR16000 M1 in JMA on a daily trial basis. Figure 5 shows an example of 72 hour forecasts made by GSM (JMA's operational forecasting), NICAM, and AGHEXA. As for this example of 12UTC Jan. 2017 initial forecasts, 500hPa geopotential height by AGHEXA was rather similar to GSM's than NICAM's.

4. Computational performances

This year (2017) the calculation domains for Yang-grid systems were reviewed and optimized, and the redundant calculation and MPI communication were reduced. The computational performance of AGHEXA was examined on K computer (RIKEN AICS), Fujitsu FX100 at the Meteorological Research Institute (MRI), and HITACHI SR16000 M1

(JMA). Results are shown in Figure 6. As is shown in the figure, the scalability of AGHEXA is improved especially for HITACHI SR16000 M1.

5. Summary

A hexahedral Yin-Yang grid model AGHEXA has been developed and examined.

Test cases by Jablonowski and Williamson (2006) and Klemp, Skamarock, and Park (2015) were examined. Results for Jablonowski and Williamson (2006) were as good as other models when using the iso-geopotential lower boundary. We still work on the supercell simulation test of Klemp, Skamarock, and Park (2015). Results will be reported at the PDEs workshop. Daily forecast results by AGHEXA have been also examined comparing with the operational forecasting and NICAM. The calculation domains of Yin-grids and MPI communication are revised,

and they improved computing performance at the three super computing systems.

Acknowledgement

The study at K computer is supported by the Japanese high performance computing infrastructure (HPCI) projects. The calculation method for Jablonowski and Williamson (2006) was proposed by Dr. Ryuji Yoshida (RIKEN AICS). Daily forecasts with NICAM have been produced by Mr. Keiichi Katayama (Japan Meteorological Agency).

REFERENCES

Baba, Y., K. Takahashi, and T. Sugimura 2010: Dynamical Core of an Atmospheric General Circulation Model on a Yin-Yang Grid. *Mon. Wea. Rev.*, **138**, 3988-4005.

Dudhia, J., and J. F. Bresch, 2002: A Global Version of the PSU-NCAR Mesoscale Model, *Mon. Wea. Rev.*, **130**, 2989 - 3007.

Ishida, J., C. Muroi, K. Kawano, and Y. Kitamura 2010: Development of a New Nonhydrostatic Model ASUCA at JMA. *CAS/JSC WGNE Research Activities in Atmospheric and Oceanic Modeling*.

Jablonowski, C., and Williamson, D. L., 2006: A baroclinic instability test case for atmospheric model dynamical cores. *Q. J. R. Meteorol. Soc.*, **132**, pp. 2943–2975.

Kageyama, A. and T. Sato 2004: "Yin-Yang grid": An overset grid in spherical geometry. *Geochem. Geophys. Geosys.* **VOL. 5**, Q09005, doi:10.1029/2004GC000734.

Klemp, J. B., W. C. Skamarock, and S.-H. Park 2015: Idealized global nonhydrostatic atmospheric test cases on a reduced-radius sphere, *J. Adv. Model. Earth Syst.*, **07**, doi:10.1002 / 2015MS000435

Peng, X., F. Xiao, and K. Takahashi 2006: Conservative Constraint for a Quasi-Uniform Overset Grid on the Sphere. *Quart. J. Roy.*

Meteor. Soc., **132**, 979-996.

Qaddouri, A., and V.Lee, 2011: The Canadian Global Environmental Multiscale model on the Yin-Yang grid system, *Quart. J. Roy. Meteor. Soc.*, **137**, 660, 1913-1926, DOI: 10.1002/qj.873.

Sakamoto, M, J. Ishida, K. Aranami, K. Kawano, K. Matsubayashi, T. Hara, H. Kusabiraki, T. Ito, H. Kurahashi, and Y. Kitamura 2015: Development of a Hexahedral Yin-Yang Grid Global Model: AGHEXA. *Proceeding of the workshop on Partial Differential Equations on the Sphere in Seoul, Korea, 19-22, Oct 2015*.

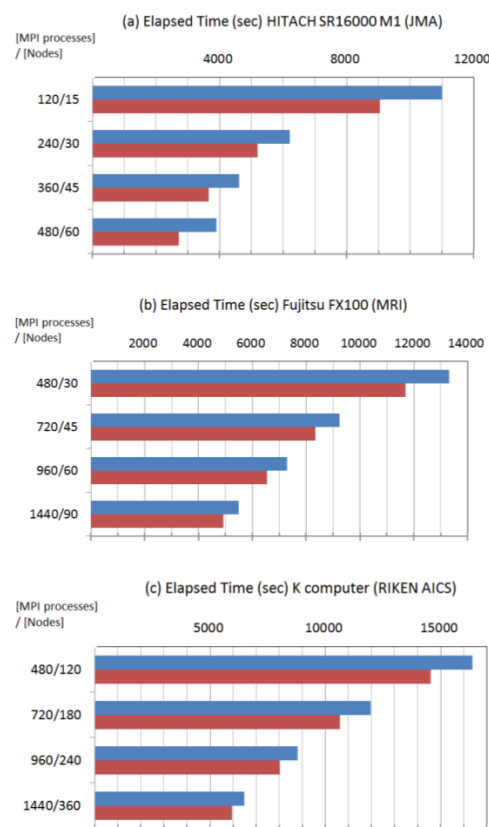


Figure 6. Elapsed times for a 0.25-deg 57-layer 84-hour prediction by AGHEXA on the three super-computer systems. Blue bars are by a version in 2015, and red ones in 2016.

Consistent 3D turbulence parametrization in circulation models

Urs Schaefer-Rolffs

We present an extension of the Dynamic Smagorinsky model (DSM) to parameterize the subgrid-scale momentum diffusion in global circulation models (GCM). In contrast to the standard approach, the test filter to determine the Smagorinsky parameter is separated from the resolution scale to exclude potential interactions. In addition, in GCMs the horizontal and vertical scales are usually treated differently due to gravity. While for the turbulent vertical diffusion of horizontal momentum a classical Smagorinsky approach is common, the respective horizontal diffusion in the free atmosphere is usually neglected. We show how to formulate the generalized DSM as subgrid-scale horizontal momentum diffusion to run stably a GCM without hyperdiffusion. Furthermore, the idea of stratified turbulence is applied to find a dynamic approach also for the vertical diffusion. Both improvements allow for a realistic spectrum of kinetic energy (almost) up to the resolution scale.

SPH-REXI: A parallel-in-time method for linear oscillatory problems with spherical harmonics

Martin Schreiber¹ and Richard Loft²

¹Harrison Building, North Park Road, Exeter, EX4 4QF, United Kingdom

²1850 Table Mesa Drive, Boulder, CO, 80305, United States

Abstract

One of the current trends in dynamical core development is to research methods which overcome inherently sequential time step restrictions. Our work is based on the rational approximation of exponential integrators (REXI). In contrast to standard time stepping methods which suffer of sequentially executed time steps, REXI allows a decomposition into a sum of independent subproblems for arbitrarily long time steps which are not restricted by the CFL. The independence of the subproblems leads to a massively parallel solver.

Applying this method for simulations on the sphere including the varying Coriolis effect results in two major challenges for the REXI subproblems: The first one is to deal with the varying Coriolis effect. Here, we present a way which allows incorporating the Coriolis effect in the linear part of the REXI approach for arbitrary discretizations on A-grids. This approach results in a Helmholtz-like formulation with the additional Helmholtz-extending term created by the space-varying Coriolis effect. The second challenge is the requirement of an efficient and highly accurate solver for each of the complex-valued Helmholtz-like subproblems. We present a solver which is based on particular features of spherical harmonics for simulations on the sphere. This solver strategy reduces the computations for each REXI subproblem to solving close-to-diagonal banded matrices in spectral SPH space. Because of this close-to-diagonal banded matrix property, this allows time-to-solution efficient utilization of complex-valued direct solvers.

In the context of HPC we discuss the performance issues and efficiency of this parallel-in-time approach. Regarding geoscience applicability, we show studies of numerical properties with selected benchmarks which are considered to be of high relevance for atmospheric and ocean simulations.

Improving accuracy over steep slopes

James Shaw, Hilary Weller, John Methven, Terry Davies

Representing terrain in atmospheric models creates mesh distortions that increase advection errors and pressure gradient errors. Finer meshes are able to resolve steep slopes that result in larger distortions and increased numerical errors. Smoothing terrain-following coordinates can help to reduce distortions, and the cut cell method reduces distortions even further, but creates arbitrarily small cut cells that can severely constrain the time-step for explicit methods. Regardless of the mesh type, distortions can only be reduced, not eliminated.

We present two new methods that improve the accuracy of flows over steep terrain on arbitrarily-structured meshes. First, the slanted cell mesh is a new method that avoids additional time-step constraints derived from a von Neumann analysis to ensure numerical stability on severely distorted meshes.

The method-of-lines advection scheme is assessed using a new test case in which a tracer placed at the ground is transported over steep mountains. We find that the scheme is largely insensitive to the type of mesh and steepness of the mountains. We also demonstrate that the scheme is second-order convergent irrespective of mesh distortions. Incorporating the new advection scheme into a dynamical core we show that, compared to terrain-following and cut cell meshes, the slanted cell mesh reduces pressure gradient errors.

Compatible finite element methods for numerical weather prediction

Jemma Shipton*, Colin Cotter

We present recent work on developing a compatible finite element model for numerical weather prediction. This work has been motivated by the requirement for numerical discretisations that are stable and accurate on nonorthogonal grids (such as icosahedral or equiangular cubed sphere grids) without sacrificing properties of conservation, balance and wave propagation that are important for accurate atmosphere modelling on the scales relevant to weather and climate (Staniforth et. al. 2012).

Compatible finite element methods are a type of mixed finite element method (where different finite element spaces are used for different fields) where the divergence of the velocity space maps on to the pressure space. This necessitates the use of div-conforming finite element spaces for velocity, such as Raviart-Thomas and Brezzi-Douglas-Marini, and discontinuous finite element spaces for pressure. The main reason for choosing compatible finite element spaces is that they have a discrete Helmholtz decomposition of the velocity space; this means that there is a clean separation between divergence-free and rotational velocity fields. Cotter and Shipton (2012) used this decomposition to demonstrate that compatible finite element discretisations for the linear shallow water equations satisfy the basic conservation, balance and wave propagation properties listed in Staniforth and Thuburn (2012).

In the talk we will show the progress we have made towards extending this approach to the fully 3D equations via 1) a spherical shallow water model and 2) a compressible vertical slice model. Many current atmospheric models use a staggered Charney Phillips grid in the vertical to ensure a good representation of hydrostatic balance. In the finite element context the equivalent staggering requires the temperature field to be discontinuous in the horizontal direction but continuous in the vertical. We present a stable and accurate advection scheme for this field. The success of this approach is illustrated by benchmarking results from our model, the Gusto dynamical core, implemented in Firedrake (www.firedrake.org/gusto).

Staniforth, A., and J. Thuburn. *Horizontal grids for global weather and climate prediction models: a review*. Quarterly Journal of the Royal Meteorological Society 138.662 (2012): 1-26.

Cotter, C. J., and J. Shipton. *Mixed finite elements for numerical weather prediction*. Journal of Computational Physics 231.21 (2012): 7076-7091.

Coupling finite difference physics parametrizations to a mixed finite element dynamical core

B.J. Shipway and the GungHo and LFRic teams

In order to meet the challenges which will be presented by the next generation of exascale supercomputers, we are developing a replacement for the Met Office Unified Model (MetUM). The new atmospheric model will seek to retain the performance of the current ENDGame dynamical core and associated subgrid physics, while also enabling a far greater scalability and flexibility to accommodate future supercomputer architectures.

The model will include a re-write of the dynamical core and a choice has been made to use a mixed finite element method, where different function spaces are used to represent the various fields. These methods preserve a number of properties desirable for an atmospheric dynamical core including: good inherent conservation, compatibility with certain continuous vector calculus operators and can be seen as extending the C-grid/Charney-Philips staggering into a finite element framework.

For the subgrid physics parametrizations, the preference is to retain (as far as possible) the finite difference formulations currently used in the MetUM. This requires a coupling to the finite element representations of the prognostic fields, which retains among other things the desirable properties upon which the dynamical core is predicated.

In this work we will provide an overview of the strategy used for the coupling and provide some early results of its application.

A Lagrangian multiscale FEM for transient passive advection-diffusion equations with strong transport

Konrad Simon* and Jörn Behrens

University of Hamburg/CliSAP, Grindelberg 5, 20144 Hamburg, Germany

Abstract. Long simulation times in climate sciences typically require coarse grids due to computational constraints. Nonetheless, unresolved subscale processes, such as (slowly) moving land-sea interfaces or ice shields, significantly influence the prognostic variables and can not be neglected for a reliable long term simulation. Nowadays, the influence of subscale processes is typically incorporated via subscale parametrizations that often use a heuristic coupling of scales.

To improve mathematical consistency in the way the upscaling is done we investigate novel bottom-up Lagrangian techniques for advection dominated problems arising in climate simulations [3]. Our tools originate from multiscale finite element methods for elliptic problems that arise, for example, in oil reservoir modeling [1, 2]. The ideas need to be modified in order to account for the transient and advection dominated character of climate models.

We aim to present several Garlerkin based ideas to account for the typical difficulties in climate simulations and to present results from sample runs for a simple advection-diffusion equation with rapidly varying coefficients. Our modified ideas employ a frequency based splitting of the advection term. Mean flow is treated in a Lagrangian setting while the oscillatory flow needs a special treatment of boundary conditions on the fine scales in an Eulerian setting.

References

- [1] Yalchin Efendiev and Thomas Y Hou. *Multiscale finite element methods: theory and applications*, volume 4. Springer Science & Business Media, 2009.
- [2] Ivan G Graham, Thomas Y Hou, Omar Lakkis, and Robert Scheichl. *Numerical analysis of multiscale problems*, volume 83. Springer Science & Business Media, 2012.
- [3] Peter H Lauritzen, Christiane Jablonowski, Mark A Taylor, and Ramachandran D Nair. *Numerical techniques for global atmospheric models*, volume 80. Springer Science & Business Media, 2011.

*konrad.simon@uni-hamburg.de

Higher-order finite volume differential operators on the icosahedral spherical grid

Christopher Subich

January 11, 2017

Abstract

The icosahedral spherical grid is a quasi-uniform horizontal discretization of the sphere, generated by successively refining the triangular faces of an embedded icosahedron. The corresponding dual grid, which consists of a twelve pentagonal cells and a large number of hexagonal cells, is a promising candidate for the spherical discretization in the dynamical cores of numerical atmospheric models.

This work develops non-staggered, finite-volume differential operators for the gradient, curl, and divergence on this semi-structured grid, capable of at least third-order convergence, through the use of higher-order reconstruction of scalar and vector function values at cell edges from their cell-average values. This reconstruction process directly incorporates the sphere's intrinsic curvature, which is necessary to achieve higher-order results.

Coupling concepts based on Schwarz domain decomposition methods

Anusha Sunkisala*, Konrad Simon and Jörn Behrens
University of Hamburg/CliSAP, Grindelberg 5, 20144 Hamburg, Germany

Abstract. In earth system models (ESM) a number of sub-components, like atmosphere, oceans, terrestrial and cryospheric systems are coupled. For long term climate simulations, the coupling mechanism is crucial for maintaining accuracy and conservation properties of the system. It can be assumed that the coupling accuracy influences the general system's sensitivity to perturbations as well as its convergence to realistic solutions. Therefore, a mathematically consistent algorithm and structure preservation of the exchanged quantities play an important role for achieving relevant simulation results.

My research focuses on adaptation of Schwarz decomposition methods to idealized ocean atmosphere coupling to obtain a stable and consistent coupling method. The idea of Schwarz decomposition methods is to separate the original problem on a domain into sub-domains (sub-problems) which can be solved separately. An iterative process is then applied to achieve convergence to the solution of the original problem.

I started with the implementation of Schwarz decomposition methods to 1D elliptic and parabolic equations as toy models to better understand these methods and to extend this type of method for atmosphere and ocean coupling.

*anusha.sunkisala@uni-hamburg.de

The Gibbs function: a route to consistent and flexible thermodynamics in atmospheric models

John Thuburn
University of Exeter

Moist thermodynamics is complicated, and atmospheric models typically make various approximations. These are not always made in a self-consistent way, which could lead to spurious sources or sinks of energy and entropy. Self-consistency can be ensured deriving all thermodynamic quantities from a thermodynamic potential such as the Gibbs function. The Gibbs function may be approximated, but any such approximations are inherited by all derived quantities in a way that guarantees self-consistency. Here, the feasibility of using the Gibbs function in an atmospheric model is demonstrated through the development of a semi-implicit, semi-Lagrangian vertical slice model, and its application to a standard buoyant bubble test case. The Gibbs function approach is very flexible, allowing different equations of state to be implemented by changing only a single subroutine. The flexibility of the approach is demonstrated by running the buoyant bubble test case with four different equations of state corresponding to dry air (perfect gas), moist air that is saturated, a pseudo-incompressible fluid, and an incompressible fluid.

A potential enstrophy and energy conserving scheme for the shallow water equations on the cubed sphere

Michael Toy, Ramachandran Nair
NCAR

The Arakawa and Lamb (1981, hereafter AL81) potential enstrophy and energy conserving scheme for the shallow water equations was formulated for orthogonal coordinate systems. As a consequence, the scheme is directly applicable on the sphere only for grids based on latitude-longitude coordinates, which have singularities at the poles, or on the cubed sphere with orthogonal grids, which typically contain regions of highly-clustered grid elements that limit the efficiency of numerical integrations due to CFL constraints. In a recent paper, Toy and Nair (2017) extended AL81 to generalized curvilinear coordinates, including non-orthogonal systems, and tested the scheme on a flat-plane geometry with doubly periodic coordinate systems. We have recently extended the scheme to allow for coordinate discontinuities, and are applying the scheme to the cubed sphere. This will allow for a broader range of grid configurations with quasi-homogeneous grid spacing. We will contrast the conservation and consistency characteristics of the new scheme to the original AL81 scheme.

References :

Arakawa, A., and V. R. Lamb, 1981 : A potential enstrophy and energy conserving scheme for the shallow water equations. *Mon. Wea. Rev.*, 109, 18–36, doi 10.1175/1520-0493(1981)109<0018 :APEAEC>2.0.CO ;2.

Toy, M. D., and R. D. Nair, 2017 : A potential enstrophy and energy conserving scheme for the shallow water equations extended to generalized curvilinear coordinates. *Mon. Wea. Rev.*, doi 10.1175/MWR-D-16-0250.1 (in press).

Adaptive shallow water wave simulations with RKDG schemes on triangular grids

Stefan Vater and Jörn Behrens

Department of Mathematics, Universität Hamburg, Germany
and
CEN – Center for Earth System Research and Sustainability,
Universität Hamburg, Germany

Due to their potentially disastrous impact on environment and society the numerical simulation of Tsunami events demands for accurate and efficient algorithms. In this talk, two important aspects in this respect are discussed: dynamic mesh adaptivity and the treatment of inundation at the coast.

The local nature of the tsunami wave promotes the application of adaptive grid algorithms. Especially in the beginning, when the tsunami has just been generated by an earthquake, an accurate solution is only needed locally. In the presented framework the adaptive grid generator *amatos* is applied, which uses triangular grids and a bisection-based refinement strategy. A local error indicator controls the adaptivity of the grid.

Another important part in the numerical modeling of tsunami or storm surge events is the accurate and robust treatment of flooding and drying at the coast. Within the framework of the shallow water equations, such an algorithm should preserve the steady state of a fluid at rest, be mass conservative and should preserve the positivity of the fluid depth. Here we present a novel treatment for second-order DG schemes which is based on a limiting approach. The core of the method is a velocity based "limiting" of the momentum, which provides stable and accurate solutions in the computation of wetting and drying events.

The performance of the method is verified by several analytical test cases and is finally applied to realistic tsunami test problems.

Third-order accurate MPDATA for arbitrary flows

Maciej Waruszewski^a, Piotr K. Smolarkiewicz^b, Christian Kühnlein^b

^aInstitute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland

^bEuropean Centre for Medium-Range Weather Forecasts, Reading, United Kingdom

We extend the structured-grid MPDATA scheme of Margolin & Smolarkiewicz (1998)—third-order-accurate for uniform flows, but second-order-accurate otherwise—to third-order accuracy for arbitrary flows. The extended scheme retains the basic iterative-upwind structure of MPDATA and, thus, the characteristic sign-preservation property of the MPDATA approach. The current extensions augment the pseudo velocities of corrective iterations with additional terms, to account for temporal and spatial variability of the flow. In order to derive these new terms we identify the leading truncation error of the basic second-order accurate MPDATA using the modified equation analysis. In the spirit of the Lax-Wendroff approach, the second and third derivatives with respect to time are expressed in terms of spatial derivatives by repeated application of the modified equation. The results of the analytical derivation are verified and further generalised using computer algebra.

This talk will highlight the main points of the derivation and discuss the key aspects of implementation with the focus on the accuracy, efficiency and stability of calculations. The extended scheme is compared with the established variants of MPDATA using time dependent deformational-flow benchmarks of Nair & Jablonowski (2008) and Lauritzen et al. (2012). Results of extensive numerical experiments verify the third-order convergence rate of the new scheme.

References

- L. Margolin, P. K. Smolarkiewicz 1998: Antidiffusive velocities for multipass donor cell advection. *SIAM Journal on Scientific Computing* 20(3) 907-929.
- R. D. Nair, C. Jablonowski 2008: Moving vortices on the sphere: A test case for horizontal advection problems. *Monthly Weather Review* 136.2 699-711.
- P. H. Lauritzen, et al. 2012: A standard test case suite for two-dimensional linear transport on the sphere. *Geoscientific Model Development* 5.3.

Optimally transported meshes on the Sphere for Global Atmospheric Modelling

Hilary Weller, Philip Browne

Numerical weather and climate predictions could be dramatically improved with the use of adaptive meshes - locally varying the spatial resolution to improve accuracy. R-adaptivity, (mesh re-distribution) is an attractive form of adaptivity since it does not involve altering the mesh connectivity, does not create load balancing problems on parallel computers, does not require mapping solutions between different meshes, does not lead to sudden changes in resolution and can be retro-fitted into existing models.

Optimal transport techniques, solving a Monge-Ampere equation, can be used to generate r-adapted meshes which are guaranteed not to tangle. I will describe the first numerical method for solving this optimal transport problem on the surface of the sphere. I will then compare numerical solutions of the Euler equations on r-adaptive meshes with those on fixed meshes.

Time Integration of Euler Equations using Dual Time-Stepping and Multigrid Methods

Tae-Hyeong Yi

Korea Institute of Atmospheric Prediction Systems, Seoul, Korea

In simulating atmospheric flows with a three-dimensional global model, numerical difficulties are encountered due to stiff spatial scales in horizontal and vertical directions. In general, the vertical grid spacing in atmospheric models is much smaller than that of the horizontal. For the explicit scheme of time integration, which enables the models to achieve higher time accuracy, therefore, the maximum time step of the atmospheric models is limited by the vertical grid spacing. One of the popular approaches for circumventing the time step limitation is to employ implicit scheme. However, this implicit scheme requires significantly higher computational effort per a time iteration than the explicit scheme, is more difficult to parallelize, and sacrifices the solution accuracy.

To obtain a large time step and reduce total computational time, a dual time-stepping scheme is adopted in this study as the framework of time integration. This is second-order backward difference for the temporal terms of governing equations. In the dual time-stepping framework, two different time steps are derived from the governing equations, which are the global physical time step used in an outer loop and the pseudo time step used in an inner loop. The major advantage of this scheme is that the physical time step is not restricted by the Courant-Friedrichs-Lewy (CFL) condition due to the stability issue. Moreover, various acceleration approaches such as local time-stepping, residual smoothing, and multigrid schemes, usually employed for steady-state problems, can be applied to the inner loop in order to improve the convergence performance. In the inner loop, the explicit multistage Runge-Kutta (RK) scheme can be used with those acceleration approaches. In addition to the explicit schemes, other implicit schemes might be employed in the inner loop. The multigrid approach, which is most powerful tool for accelerating the convergence, is based on a series of the solutions on successively coarser grids. This denotes the continuous cycles of the restriction to coarser grids and the prolongation to finer grids. In this study, the Full Approximation Storage (FAS) scheme of the multigrid, which was developed for nonlinear equations, is employed with the explicit third-order, three-stage RK method for two-dimensional compressible Euler equations. Moreover, the local time-stepping and implicit residual smoothing approaches are combined with the multigrid together.

In order to evaluate the convergence performance of the multigrid scheme in the framework of the dual time-stepping, the comparison of the wall-clock time and normalized error norm is carried out with various test cases where the baseline scheme of time integration is set to the explicit third-order, three-stage RK method.